Enter, En04-A-v3, 2026, pp. 1-45

En04-A Under the Enter Philosophy—Readout, Recovery, and Residual Response: Magic, Complete Complements, and the Limits of a Geometric Interpretation, with a Layer of Conditional Predictions

Ngo Cheung (authorship requires confirmation)

Version 3 (conditional predictions); parent: En04-A-v2

Abstract

We study a finite, already-prepared quantum carrier V: C → R ⊗ B and ask when its complete residual B responds to the logical code. Exact recovery from R is equivalent to a constant complete complement. Non-Clifford resources neither force nor are forced by such response: a Clifford copier responds, while a local non-Clifford gate and a fixed syndrome with bipartite non-local magic do not. For a controlled-rotation family, δ_full(θ) = (sin θ)/2, ε_rec^R(θ) = (1 − cos θ)/2, 0 ≤ θ ≤ π/2. A first-order commutator witness is sufficient for response, and a Pauli test decides constancy for Clifford encoders. Readout-and-drop is partial trace, and classical records are not a quantum decoder. Version 2 adds eight theorems that turn these separations into criteria a proposal can fail. Residual response never forces a geometric factor, with the exact scope on a fixed residual. Each hypothesis of the conditional bridge is necessary for a magic-accompanied conclusion, while the resource certificate is idle for the printed one. Leakage is not a function of a Clifford flag, a T-count, or a stabilizer-entropy bin, and the worst-case response-indicator error is 1/2. A coupling with vanishing first-order witness has δ_full = 1/2sin^2 λ. Equal leakage occurs with different resources, and equal resource class with different leakage. Channel data do not select a heat ledger; a leakage history does not fix an entropy, record, or Page history; and residual readout followed by drop returns exactly the radiation channel, with the classical-record bound 1 − 1/d kept distinct from 1 − d^{−2} and from clock constants. Version 3 adds a conditional-prediction layer (Section). Sixteen proposed cosmological ``predictions" become theorems of the form if a package supplies these data, then this holds on the finite carrier, each with a witness and a non-claim. The leakage–response bound holds with the sharp constant, |Δ⟨O⟩| ≤ 2s(O)δ_full ≤ 4‖O‖δ_full with s(O) the spectral spread, equivalently γ_O ≤ 2δ_full, with equality for the copier; the proposed constant 2‖O‖ is false. A constant supplied profile has scalar compression and is blind for every carrier, while a nonconstant one gives the sampling witness (κ_1 − κ_0)sin^2 η + sin 2ηRe c with an input-independent host. Both marginals can be constant while a product observable responds with the optimal value 2. One host datum allows every leakage in [0, 1 − d^{−2}]; a code-dependent qubit clock leaks 1/2√(1 − |⟨τ_0|τ_1⟩|^2); an echo has F_e = d^{−2} yet estimates its parameter; the classical floor 1 − 1/d stays distinct from 1 − d^{−2}; and a coefficient enters only through a supplied sensitivity law. No layer of the five-layer programme implies the next, each arrow of the constraint chain needs an added law, and the eight candidate phenomena remain conjectures with proved finite analogues. No cosmological prediction is made. Standing ledger. No metric, field-to-memory receiver, admitted analytic quantum coefficient, or gravitational backreaction theorem is constructed. O.1 stays NOT_CLOSED; no quantum coefficient is admitted; XV-B's admission exit is not opened; and the separate XVI-N obligations are not discharged.

Bibliographic record

Title	En04-A Under the Enter Philosophy—Readout, Recovery, and Residual Response: Magic, Complete Complements, and the Limits of a Geometric Interpretation, with a Layer of Conditional Predictions
Author	confirmation), Ngo Cheung (authorship requires
Publication date	2026
Institution	Enter

Report number En04-A-v3
Publisher Cheung Ngo Medical Limited
First page 1
Last page 45
Language en

Citations identify source records only; no proof transfer. No quantum coefficient is admitted. Gate 0 and ST4b stay NOT_SUPPLIED; U6-sel is not moved; T3 stays unresolved; no Host C is introduced; no numerical value of κ is given.

The corrected logical structure.

$$\begin{gathered}\text{non-Clifford resource} \;\Rightarrow\; ?\text{residual response},\\ \text{Clifford encoding} \;\Rightarrow\; ?\text{constant complete complement},\\ \text{constant complete complement} \;\Leftrightarrow\; ?\text{exact whole-code recovery from} R,\\ \text{residual response} \;\Rightarrow\; ?\text{physical geometric backreaction}.\end{gathered}$$

A responding non-Clifford model can be proved after its coupling, code, preparation, access partition, and response witness are fixed. It cannot be inferred merely from the word ``magic."

Introduction and the corrected problem

The operational question addressed here begins after a preparation has produced a finite quantum carrier. It concerns a complete encoding or evolution map, an accessible output, an inaccessible residual, a quantum decoder, and a separate destructive observation.

The proposed slogan

``Non-local magic enables geometry to respond to matter"

contains several logically distinct statements. In the present channel formulation, the relevant operational condition is non-constancy of the complete residual channel. However, neither ordinary non-Cliffordness nor a generic notion of non-local magic is equivalent to this condition. Moreover, an inaccessible quantum memory is not automatically a geometric degree of freedom.

Exact quantum correction and no-hiding supply the recovery criterion [KnillLaflamme,BraunsteinPati]. Magic belongs to a resource theory relative to a specified stabilizer structure [VeitchEtAl]. These are different mathematical structures.

A related but more specialized literature studies nontrivial area operators and perturbative matter–geometry correlations in holographic or skewed subsystem codes [Cao2024,CaoEtAl2026]. Its hypotheses, subsystem decompositions, and magic quantities are not identical to a generic channel $C \to RB$. Section explains the separation.

Relation to the Cheung manuscripts

Landmark I gives a Born-law and positive-branch residual characterization inside its declared strategy class [CheungI]. En03-A supplies the minimal linear interface and ownership discipline [CheungEn03A]. We define the finite fragment locally and use the supplied characterization only at its stated documentary level.

Hawk04-A supplies complete preservation, the global restore map, the exact complementary-channel criterion, the precise no-hiding conclusion, and the four-Majorana control [CheungHawk04A]. Hawk04-B distinguishes quantum return, destructive observation, complete apparatus outputs, and operational discard [CheungHawk04B]. The statements needed below are proved locally.

Hawk02-E supplies the regulated microscopic architecture and the separation of source energy from residual independence [CheungHawk02E]. The complete-process and apparatus disciplines of En02-B, En02-C, En02-E, and Em03-C are maintained [CheungEn02B,CheungEn02C,CheungEn02E,CheungEm03C]. Availability of a decoder, existence of a channel, and executable implementation under a resource budget remain distinct.

The XIV/XV/XVI and OpenAI catalogue comparisons are documentary separations only. Citations identify source

records only; no proof transfer. They do not supply a premise for any channel, magic, or response theorem in this paper.

Four repairs made before completing the plan

• **Magic is not leakage.** A non-Clifford gate can act entirely within the recoverable logical sector. Conversely, a Clifford interaction can copy a basis label to the residual.

• **The restore map does not move the diary into the residual.** It recovers the diary from the complete output *RB*. Residual-only recovery follows when the radiation channel is constant, not merely from global preservation.

• **Readout-and-drop is not permanent physical deletion.** It is a trace in the observer's channel semantics. Its complete unitary dilation retains omitted apparatus outputs.

• **Residual response is not yet geometric response.** One must identify actual geometric observables and show that they distinguish the declared matter inputs in the same physical model.

The completed result is consequently a corrected operational bridge with explicit witnesses and counterexamples, not a proof of the original unconditional slogan.

What is new in version 2

Section adds Theorems , , , , , , , and , with Lemmas and and Proposition . Every result now ends with a non-claim. Appendix gives the v1 →v2 crosswalk; Appendix records the verdict on each planned theorem and the plan corrections; Appendix summarizes the finite numerical checks. No label of version 1 is removed or renamed.

What is new in version 3

Section states each of sixteen proposed predictions as a conditional theorem on declared packages (Definition), with explicit witnesses and exact minimax errors (Lemma); it adds the five-layer proposition (Proposition), the constraint-chain theorem (Theorem), the discovery requirements (Definition), and labelled conjectures with finite analogues (Conjecture , Proposition). Layers above the probe use only prescribed hosts and supplied functions; no dynamics is constructed. Appendix gives the v2 →v3 crosswalk and Appendix the verdicts, plan corrections PC-10 to PC-17, and documentary corrections. No label of version 1 or 2 is removed or renamed.

Complete channels and residual response

Let $C \simeq \mathbb{C}\text{^}d$, $d \geq 2$, be a finite logical code. Output Hilbert spaces are separable. Channels act normally on trace-class operators and are completely positive and trace preserving, abbreviated CPTP.

For an isometry

$$V : C \to R \otimes B,$$

set

$$\mathscr{V}(X) = VXV^{\dagger}, \qquad \mathscr{M}(X) = \mathrm{Tr}_B\, \mathscr{V}(X), \qquad \mathscr{C}(X) = \mathrm{Tr}_R\, \mathscr{V}(X).$$

The residual B includes every inaccessible output: preparation purifiers, retained memories, uncollected modes, displaced carriers, and inaccessible apparatus factors.

Distances are

$$d_{\mathrm{tr}}(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1, \qquad d_{\diamond}(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_{\diamond}.$$

All channel statements retain arbitrary untouched references. For finite input, standard diamond-norm and Choi conventions are as in [Watrous].

Define the replacement channel

$$\mathscr{T}_{\sigma}(X) = \mathrm{Tr}\,(X)\sigma$$

and the quantities

$$\varepsilon^{R}_{\mathrm{rec}}(V) = \inf_{\mathscr{D} : R \to C \text{ CPTP}} d_{\diamond}(\mathscr{D}\mathscr{M}, \mathrm{id}_C),$$

$$\delta_{\mathrm{full}}(V) = \inf_{\mathrm{sigmad}} _\diamond(\mathscr{C}, \mathscr{T}_{\sigma}).$$

Definition. The complete residual responds to the logical code if $\mathscr{C}$ is not a replacement channel. It is nonresponding if $\mathscr{C} = \mathscr{T}_{\sigma}$ for one fixed state σ.

This concerns variation with the logical input at a fixed encoding or fixed time. Dependence of a fixed replacement state σ_{λ} on an external coupling parameter λ is not, by itself, logical-input response.

No ``rigid geometry'' is defined by Cliffordness. When convenient, we call a *proved nonresponding interface* an operationally rigid interface. This terminology carries no metric assertion.

Lemma. For isometries into a common output space,

$$d_{\diamond}(\mathscr{V}_1, \mathscr{V}_2) \le \|V_1 - V_2\|.$$

The same allowance holds after a common partial trace. CPTP pre- and postprocessing contract half-diamond distance.

On fixed output spaces, recovery error and distance to the replacement-channel set are one-Lipschitz:

$$|\varepsilon_{\mathrm{rec}}(\mathscr{N}_1) - \varepsilon_{\mathrm{rec}}(\mathscr{N}_2)| \mathrm{led}_{\diamond}(\mathscr{N}_1, \mathscr{N}_2),$$

$$|\delta_{\mathrm{full}}(\mathscr{C}_1) - \delta_{\mathrm{full}}(\mathscr{C}_2)| \mathrm{led}_{\diamond}(\mathscr{C}_1, \mathscr{C}_2).$$

Proof. For normalized vectors u, v,

$$d_{\mathrm{tr}}(|u\rangle\langle u|, |v\rangle\langle v|) = \sqrt{1 - |\langle u, v\rangle|^2} \le \|u - v\|.$$

Apply this to referenced isometric outputs, then use convexity and partial-trace contraction.

For each decoder, the triangle inequality changes its error by at most the channel comparison allowance. Taking infima in both directions proves recovery stability. The same argument applies to distance from the replacement-channel set.

par *Non-claim.* These are channel-distance facts; no dynamical estimate is claimed. par

Lemma. Put $a_d = 1 - d^{-2}$. Then

$$\inf_{\mathrm{sigmad}} _{\diamond}(\mathrm{id}_C, \mathscr{T}_{\sigma}) = a_d.$$

Every channel from C has recovery error at most a_d and leakage distance from replacement channels at most a_d .

Proof. For any replacement state, the Bell-input output is $I_Q/d \otimes \sigma$. Its overlap with the target Bell projector is d^{-2}, giving the lower bound a_d.

The uniform twirl over the d^2 Weyl unitaries is $\mathscr{T}_{I_C/d}$. Their Bell outputs are mutually orthogonal. The triangle inequality and that Bell input give the matching distance a_d.

A decoder which ignores its input and prepares I_C/d gives the recovery upper bound. For a channel Φ,

$$\Phi \mathscr{T}_{I_C/d} = \mathscr{T}_{\Phi(I_C/d)}.$$

Postprocessing contraction gives the leakage upper bound.

par *Non-claim.* The constant a_d is a universal channel bound, not a physical error rate. par

Preservation, exact recovery, and response witnesses

Theorem. Let $J: C \to H_{in}$ be an isometry, let $|\Omega\rangle$ be a fixed normalized ancilla, and let U be unitary on the complete input space. Then

$$V|\psi\rangle = U(J|\psi\rangle \otimes |\Omega\rangle)$$

is an isometry. Its complete channel has the CPTP left inverse

$$\mathscr{L}(Y) = V^{\dagger} Y V + \mathrm{Tr}\,((I - VV^{\dagger})Y)\tau_C, \qquad \tau_C = I_C/d.$$

For every code–reference trace-class operator X,

$$\|(V \otimes I)X(V^{\dagger} \otimes I)\|_1 = \|X\|_1.$$

Proof. The preparation and evolution compose to an isometry. Both terms of are completely positive; their traces sum to *Tr Y*. The second term vanishes on the encoded range, and $\mathscr{L}(VXV\^{}\{\dagger\}) = X$.

An isometric embedding preserves nonzero singular values, including after tensoring with a reference identity.

par *Non-claim.* Global restore recovers *C* from *RB*; it neither localizes the code in *B* nor deletes anything. par

The map $\mathscr{L}$ returns *C* from *RB*. It neither deletes the code nor asserts that *B* alone carries a recoverable copy. This is the restore structure in [CheungHawk04A].

Theorem. For $V: C \to R \otimes B$, the following are equivalent:

• one CPTP decoder on *R* exactly recovers the entire code;

• $\mathscr{C} = \mathscr{T}_\{\sigma\}$ for one residual state σ;

• for every code matrix unit $E_ij = |i\rangle\langle j|$,

$$\mathscr{C}(E_{ij}) = \delta_{ij}\sigma;$$

•

$$J_{\mathrm{n}}(\mathscr{C}) = \frac{I_Q}{d} \otimes \sigma;$$

• for every bounded residual observable *O*,

$$V^{\dagger}(I_R \otimes O)V \quad \text{is a scalar multiple of} I_C.$$

Here

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_i |i\rangle_Q |i\rangle_C, \qquad J_{\mathrm{n}}(\Phi) = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|).$$

When these conditions hold, there exist a Hilbert space *K*, a state σ_K, and an isometry $A: C \otimes K \to R$ such that

$$\mathscr{N}(X) = A(X \otimes \sigma_K)A^{\dagger}.$$

An exact complete decoder is

$$\mathscr{D}(Y) = \mathrm{Tr}_K\,(A^{\dagger}YA) + \mathrm{Tr}\,((I - AA^{\dagger})Y)\tau_C.$$

Proof. Linearity and expansion of the Bell projector give equivalence of (ii)–(iv). Duality gives

$$\mathrm{Tr}\,(O\mathscr{C}(X)) = \mathrm{Tr}\,(V^{\dagger}(I \otimes O)VX).$$

Thus (ii) implies (v). Conversely, in (v), evaluate at τ_C to identify the scalar as $Tr\ (O\mathscr{C}(\tau_C))$. Since bounded observables separate trace-class operators, $\mathscr{C}(X) = Tr\ (X)\mathscr{C}(\tau_C)$, proving (ii).

Assume (ii), and diagonalize

$$\sigma = \sum_{\alpha} p_{\alpha}|b_{\alpha}\rangle\langle b_{\alpha}|, \qquad p_{\alpha} > 0.$$

The support condition permits the expansion

$$V|i\rangle = \sum_{\alpha} \sqrt{p_{\alpha}}\,|r_{i\alpha}\rangle|b_{\alpha}\rangle.$$

Constancy on every matrix unit gives

$$\langle r_{j\beta}, r_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

Consequently $A(|i\rangle|\alpha\rangle) = |r_\{i\alpha\}\rangle$ is an isometry, and $\sigma_K = \sum_\{\alpha\}\ p_\{\alpha\}|\alpha\rangle\langle\alpha|$ gives . Formula is CPTP and recovers every matrix unit.

Conversely, dilate an exact decoder by an isometry $L: R \to C \otimes G$. Stinespring dilation is the standard channel structure theorem [Watrous,KSW]. The map $Z = (L \otimes I_B)V$ has

$$Z|i\rangle = |i\rangle \otimes |\eta_i\rangle,$$

because the decoded marginal is pure. Exact recovery of E_ij forces $\langle\eta_j, \eta_i\rangle = 1$. All η_i coincide. Tracing the decoded register and *G* therefore gives (iii).

par *Non-claim.* No new correction theorem is claimed; this is the exact correction and no-hiding criterion in the present conventions. par

These are the exact correction/no-hiding conditions [KnillLaflamme,BraunsteinPati] used in [CheungHawk04A]. The local proof additionally identifies the residual-observable test used below.

Corollary. If $\mathscr{N}$ is constant, B exactly recovers the code. If $\mathscr{C}$ is constant, R exactly recovers it. Both complementary outputs cannot independently recover the entire unknown code when $d \geq 2$.

Proof. Interchange R and B in Theorem . An exactly recoverable output has constant complement, which cannot itself recover two distinct inputs.

par *Non-claim.* No statement about approximate or partial hiding is made. par

Lemma. For a finite input, $\delta_full = 0$ if and only if $\mathscr{C}$ is constant. If two code states satisfy

$$x = d_{\mathrm{tr}}(\mathscr{C}(\rho_0), \mathscr{C}(\rho_1)) > 0,$$

then

$$\delta_{\mathrm{full}} \geq x/2.$$

If $O = O\^{}\{\dagger\}$, $\|O\| \leq 1$, and

$$w = |\mathrm{Tr}\,(O\mathscr{C}(\rho_0 - \rho_1))|,$$

then

$$\delta_{\mathrm{full}} \geq w/4.$$

Proof. For every replacement state, the triangle inequality gives

$$x \leq d_{\mathrm{tr}}(\mathscr{C}(\rho_0), \sigma) + d_{\mathrm{tr}}(\sigma, \mathscr{C}(\rho_1)) \leq 2d_{\diamond}(\mathscr{C}, \mathscr{T}_{\sigma}).$$

Also $w \leq \|\mathscr{C}(\rho_0 - \rho_1)\|_1 = 2x$.

If leakage is zero, choose σ_n with $d_\{\diamond\}(\mathscr{C}, \mathscr{T}_\{\sigma_n\}) \to 0$. Evaluation at τ_C gives $\sigma_n \to \mathscr{C}(\tau_C)$ in trace norm. Replacement-channel distance equals the corresponding state distance, so $\mathscr{C} = \mathscr{T}_\{\mathscr{C}(\tau_C)\}$.

par *Non-claim.* A positive witness does not locate the whole code in B. par

Remark. A positive witness proves that the residual distinguishes some logical inputs. It does not prove residual-only recovery of the entire code. Joint recovery is guaranteed by the complete isometry; local recovery requires its own theorem.

Typed preparation, coherent programs, and Enter

The declared finite fragment

Types are

$$T ::= \mathrm{I} \mid \mathrm{Q} \mid \mathrm{Bit} \mid T \otimes T,$$

with observable algebras

$$\mathscr{A}_{\mathrm{I}} = \mathbb{C}, \qquad \mathscr{A}_{\mathrm{Q}} = M_2(\mathbb{C}), \qquad \mathscr{A}_{\mathrm{Bit}} = \mathbb{C} \oplus \mathbb{C}.$$

Classical registers are block diagonal. No conversion from a quantum register to a classical register is implicit.

A judgement $P: T \to T\^{}\{'\}$ consumes the entire input interface and returns its declared output interface. Sequential and tensor composition require matching types. A live quantum name is not duplicated or silently omitted. This follows the minimal ownership reading of [CheungEn03A], rather than an unprovided larger calculus.

The primitives are pure allocation, specified finite unitaries, destructive computational readout, classical blockwise control, and classical discard:

$$\mathrm{new} : \mathrm{I} \to \mathrm{Q}, \qquad \mathrm{Enter} : \mathrm{Q}^{\otimes s} \to \mathrm{Bit}^{\otimes s}, \qquad \mathrm{dropBit} : \mathrm{Bit} \to \mathrm{I}.$$

A specified unitary constant is a semantic object, not an assertion of exact finite-gate compilation, efficiency, or hardware realization.

For a measured subsystem S and surviving quantum subsystem T, write

$$M_x = \langle x|_S \otimes I_T.$$

The destructive readout instrument is

$$\mathscr{E}_S(\rho) = \sum_x |x\rangle\langle x|_K \otimes M_x \rho M_x^\dagger.$$

The measured names are consumed. For a pure input,

$$p_x = \|M_x\psi\|^2, \qquad \psi_x = M_x\psi/\sqrt{p_x} \quad (p_x > 0).$$

No normalized residual is assigned to a zero-probability branch.

These are the destructive Born–Lüders residual formulas supplied by Landmark I within its stated strategy class [CheungI]. We define the instrument directly; we do not reconstruct a measurement-uniqueness theorem from incomplete hypotheses.

Theorem. Every finite acyclic legal program denotes a CPTP hybrid-output channel and has path maps *K_h* satisfying

$$\sum_h K_h^\dagger K_h = I.$$

Reported outcomes group the unnormalized branch maps $X \mapsto K_hXK_h\^{\dagger}$. The same statement holds with arbitrary untouched references. Every branch terminates within the finite height of its instruction tree.

Proof. Allocation and unitary gates are isometric. Readout satisfies $\sum_x M_x\^{\dagger}M_x = I$. Classical discard sums blocks without conditional normalization.

Backward induction on the instruction tree proves completeness. Sequential composition composes the path maps; coarse-graining adds their CP maps. Tensoring each path map with a reference identity proves the complete-channel statement. A finite acyclic tree has finite maximum branch length.

par *Non-claim.* No compilation, efficiency, or hardware realization is asserted. par

This is the local version of the complete-history accounting in [CheungEn02B] and [CheungEn02C]. A selected success branch is not substituted for a deterministic decoder.

Complete coherent body and local quantum return

After a carrier dictionary exists, the coherent body has type

$$P_{\text{coh}} : Q_C \to Q_R \otimes Q_B$$

and form

a := new;?(C,a) := EncodeAndPrepare(C,a);?(R,B) := CoherentBody(C,a);?returnQ(R,B);

It contains no Enter.

A local decoder has type $P_\{\mathscr{D}\}: Q_R \to Q_C\^{out}$. A finite CPTP decoder can be implemented semantically by

w := new;?(Cout,G) := DecoderUnitary(R,w);?y := Enter(G);?dropBit(y);?returnQ(Cout);

The Kraus isometry

$$L|\psi\rangle = \sum_j D_j|\psi\rangle|j\rangle_G$$

has a finite unitary extension after allocation. Readout and discard of *G* give $\sum_j D_jXD_j\^{\dagger}$. The diary output remains quantum.

Applying this decoder locally to the coherent output yields a joint program returning (Cout,B). The observer controlling only Cout sees $\mathscr{D}\mathscr{N}$. A radiation marginal is therefore a declared access restriction, not a rule silently deleting a live residual wire.

For non-power-of-two dimensions, all logical spaces are subspaces of padded qubit registers. Physical channels are completed on invalid padded inputs, for example by fixed-state preparation. Such padding does not establish a physical field-to-qubit receiver.

Optional terminal observation

A prescribed terminal measurement may be appended after quantum return:

Cout := BasisUnitary(Cout);?x := Enter(Cout);?returnBit(x);

Its target is the measurement channel $\mathscr{M}$, not *id_C*. By contraction,

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \varepsilon \quad \Rightarrow \quad d_\diamond(\mathscr{M}\mathscr{D}\mathscr{N}, \mathscr{M}) \le \varepsilon.$$

The converse for one chosen measurement fails: computational dephasing preserves its computational record but not off-diagonal matrix units.

Enter is not a magic evaluator. A program may be mathematically analysed for nonstabilizerness, but the primitive semantics of *Enter* is the instrument .

Stabilizer structure and why the proposed implication fails

The resource convention must be fixed

Fix a qubit tensor decomposition and its Pauli frame. A Clifford unitary normalizes the Pauli group. A pure stabilizer state is a Clifford image of a computational basis state. The convex hull of such states is denoted *STAB*. A state outside this convex hull has stabilizer magic in this declared convention [VeitchEtAl].

The classification concerns operations and prepared states, not a promise that every arbitrary diary input is a stabilizer state. In particular, a Clifford program can process an unknown nonstabilizer diary.

Bosonic Gaussianity, fermionic Gaussianity, and qubit Cliffordness are not identified. An arbitrary qubit dictionary can preserve channel recovery while changing the Pauli-frame description. A magic claim must consequently fix its dictionary before evaluation.

Proposition. For

$$V' = (U_R \otimes U_B)V$$

with local output unitaries,

$$\delta_{\mathrm{full}}(V') = \delta_{\mathrm{full}}(V), \qquad \varepsilon^R_{\mathrm{rec}}(V') = \varepsilon^R_{\mathrm{rec}}(V).$$

These unitaries may be non-Clifford.

Proof. The residual channel is $\mathscr{C}\{'\} = Ad_U_B\ \mathscr{C}$. Replacement states are carried bijectively to their conjugates, and unitary channel distances are invariant. On radiation, a decoder may undo *U_R*, and conversely may prepend it when comparing the two tasks.

par *Non-claim.* Output unitaries mixing *R* and *B* are not covered. par

An output unitary mixing *R* and *B* is not covered: it changes the accessible task.

Clifford leakage and non-Clifford nonresponse

Proposition. There is a Clifford coherent program with

$$\delta_{\mathrm{full}} = \varepsilon^R_{\mathrm{rec}} = \frac{1}{2}.$$

There is a non-Clifford coherent program with

$$\delta_{\mathrm{full}} = \varepsilon^R_{\mathrm{rec}} = 0.$$

Proof. For the first example, allocate *B* in $|0\rangle$ and apply *CNOT_{C → B}*, returning the original carrier as *R*. Then

$$V_{\mathrm{copy}}|i\rangle = |i\rangle_R|i\rangle_B,$$

and both marginals are computational dephasing

$$\Delta_Z(\rho) = \frac{1}{2}(\rho + Z\rho Z).$$

The two residual basis outputs are orthogonal, giving *δ_full ≥ 1/2*. The replacement *I_2/2* gives the matching upper bound: for every referenced input the two diagonal reference blocks multiply output differences of trace

norm one. Thus the half-diamond distance is at most *1/2*.

The inputs $|+\rangle$ and $|-\rangle$ have identical radiation outputs. A decoder of error ε therefore satisfies $1 \le 2\varepsilon$. The identity decoder has error *1/2*, proving the recovery formula.

For the second example, let

$$T = \mathrm{diag}\,(1, e^{i\pi/4}), \qquad V_T|\psi\rangle = T|\psi\rangle_R \otimes |0\rangle_B.$$

The complement is *𝒯_{kb 00}*, and *T^{†}* recovers exactly. The gate is non-Clifford because *TXT^{†} = (X + Y)/√(2)*, which is not a Pauli. For input $|+\rangle$, the output has Bloch components *(1/√(2), 1/√(2), 0)*, outside the convex hull of the six Pauli eigenstates.

par *Non-claim.* The CNOT copies a basis label, not an unknown state; no cloning is claimed. par

The CNOT copies a basis label, not an arbitrary unknown quantum state. There is no violation of no-cloning.

The Choi state of *V_copy* is

$$\frac{|000\rangle + |111\rangle}{\sqrt{2}},$$

a stabilizer state. Thus its recovery obstruction does not require magic in the encoding Choi state.

Even bipartite non-local magic need not cause response

To make ``non-local'' precise for the next counterexample, define a faithful pure-state indicator across a fixed bipartition:

$$D_{\mathrm{NL}}^{A:B}(\psi) = \min_{\text{substack } U_A, U_B}$$

It is zero exactly when the state is locally unitary-equivalent to a pure stabilizer state. We use it only as an indicator: no monotonicity under arbitrary local operations is asserted. Related non-local magic constructions distinguish removable local magic from magic in correlations [Cao2024,CaoEtAl2026].

Lemma. Every reduced density matrix of a pure qubit stabilizer state is proportional to a projector. Its nonzero eigenvalues are equal, and its rank is a power of two.

Proof. For an *n*-qubit pure stabilizer state with stabilizer group *𝒮*,

$$|s\rangle\langle s| = 2^{-n} \sum_{P \in \mathcal{S}} P.$$

Tracing the complementary subsystem leaves only Paulis supported on the retained subsystem *A*:

$$\rho_A = 2^{-n_A} \sum_{P_A \in \mathcal{S}_A} P_A.$$

If *𝒮_A* has *2^r* elements, its group average *2^{−r}∑_{P_A ∈ 𝒮_A} P_A* is a projector. Therefore *ρ_A* has flat nonzero eigenvalues *2^{r − n_A}* and rank *2^{n_A − r}*.

par *Non-claim.* Standard stabilizer structure; no priority is claimed. par

This standard stabilizer structure is also discussed in [FattalEtAl]; the needed statement is proved above.

Theorem. Let *0 < κ < π/4*, and prepare the fixed syndrome

$$|\chi_\kappa\rangle_{R_1B} = \cos\kappa|00\rangle + \sin\kappa|11\rangle.$$

Define

$$V_\kappa|\psi\rangle = |\psi\rangle_{R_0} \otimes |\chi_\kappa\rangle_{R_1B}, \qquad R = R_0R_1.$$

Then

$$\mathscr{C}_\kappa = \mathscr{T}_{\sigma_\kappa}, \qquad \sigma_\kappa = \mathrm{diag}\,(\cos^2\kappa, \sin^2\kappa),$$

so *δ_full = ε_rec^R = 0*.

Nevertheless,

$$D_{\mathrm{NL}}^{R_1 : B}(\chi_\kappa) > 0,$$

and the complete encoding Choi state has

$$D_{\mathrm{NL}}^{QR : B}((I_Q \otimes V_\kappa)\Phi_d) > 0.$$

Proof. Tracing *R_0R_1* gives the displayed replacement channel. Discarding *R_1* and returning *R_0* recovers the code exactly.

The two nonzero eigenvalues of the *B*-reduction are unequal. Local unitaries preserve them, whereas a pure stabilizer reduction must have a flat nonzero spectrum by Lemma . Thus no local unitaries convert *χ_{κ}* to a stabilizer state. The local unitary groups are compact and the stabilizer set is finite, so the minimum distance is strictly positive.

The Choi state is *|Φ_d⟩_QR_0 ⊗ |χ_{κ}⟩_R_1B*. For the Choi magic statement, take a qubit code, or more generally *d = 2^k*, with its fixed qubit structure. Across *QR: B*, its *B*-reduction has the same two unequal eigenvalues. The same argument applies.

par *Non-claim. D_NL* is used as an indicator, not a monotone; the tripartite quantity of [CaoEtAl2026] is not evaluated. par

For the two-qubit syndrome, the indicator can be evaluated:

$$D_{\mathrm{NL}}^{R_1 : B}(\chi_\kappa) = \sqrt{1 - \max\left\{\cos^2\kappa, \frac{(\cos\kappa + \sin\kappa)^2}{2}\right\}}.$$

Indeed, the largest overlap with a product state is *cos^2 κ*, whereas the largest overlap with a maximally entangled two-qubit state is *(cos κ + sin κ)^2/2*. These are the two possible Schmidt spectra of a two-qubit pure stabilizer state. Both overlaps are strictly below one for *0 < κ < π/4*.

This counterexample concerns bipartite non-local magic, including in a complete Choi state. It does not identify that quantity with the specialized perturbative tripartite quantity of [CaoEtAl2026]. The fixed syndrome carries no logical-input response.

Corollary. Under the generic prepared-carrier assumptions, neither

$$\text{non-Clifford program} \Rightarrow \delta_{\mathrm{full}} > 0$$

nor

$$\text{positive bipartite non-local magic} \Rightarrow \delta_{\mathrm{full}} > 0$$

is valid. Conversely, *δ_full > 0* does not imply magic.

Proof. Use Proposition and Theorem .

par *Non-claim.* Restricted resource theories with additional matter–geometry structure are not addressed. par

A genuine Clifford criterion and a responding deformation

Pauli compression for a Clifford encoder

Let *C* contain *k* logical qubits and let *A* contain *m* allocated ancilla qubits in *|0^m⟩*. A Clifford unitary *U* gives

$$V|\psi\rangle = U(|\psi\rangle_C|0^m\rangle_A),$$

with a declared output partition *RB*.

Theorem. For each Hermitian residual Pauli *P_B*, write

$$U^\dagger(I_R \otimes P_B)U = s\,P_C \otimes P_A, \qquad s \in \{-1, 1\}.$$

The complete complement is constant if and only if every such Pauli satisfies one of:

• *P_A* has an *X* or *Y* factor, so *⟨0^mrvertP_A |0^m⟩ = 0*;

• *P_C = I_C*.

Proof. Compression gives

$$V^{\dagger}(I_R \otimes P_B)V = s\,\langle 0^m \mathrm{rvert} P_A\, |0^m\rangle\, P_C.$$

The ancilla expectation is zero when an X or Y factor is present, and otherwise is one with the phase carried by s. Thus every compression is scalar precisely under the stated condition.

Residual Paulis form an operator basis. The criterion for all of them is therefore equivalent to the observable criterion in Theorem .

par *Non-claim.* Cliffordness makes the test algebraic; it does not fix its answer. par

Cliffordness makes this test algebraic; it does not predetermine its answer. The CNOT copying example fails the test because the residual Z compresses to logical Z.

A nonresponding Clifford baseline is

$$V_0|\psi\rangle = |\psi\rangle_R|0\rangle_B.$$

A less trivial baseline prepares a Bell syndrome on *R_1B* while keeping *R_0 = C*. Then B is fixed in *I_2/2*, even though it is entangled with a radiation syndrome. Constancy does not require a product output or a pure residual.

An exact non-Clifford responding program

On one logical qubit and one allocated residual qubit, define

$$R_y(2\theta) = e^{-\mathrm{i}\theta Y},$$

$$U_\theta = \mathrm{kb}\,00 \otimes I + \mathrm{kb}\,11 \otimes R_y(2\theta), \qquad 0 \le \theta \le \frac{\pi}{2}.$$

Initialize B in $|0\rangle$ and return the first qubit as R. Then

$$V_\theta|0\rangle = |0\rangle_R|0\rangle_B, \qquad V_\theta|1\rangle = |1\rangle_R|b_\theta\rangle_B, \quad |b_\theta\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle.$$

The typed body is

b := new;?(R,B) := ControlledRy_theta(C,b);?returnQ(R,B);

It is a finite semantic program with a specified unitary constant. No exact compilation into a fixed finite gate library is asserted.

Theorem. For ,

$$\mathscr{N}_\theta = \Delta_{\cos\theta}, \qquad \Delta_c(\rho) = \begin{pmatrix} \rho_{00} & c\rho_{01} \\ c\rho_{10} & \rho_{11} \end{pmatrix},$$

and

$$\mathscr{C}_\theta(\rho) = \rho_{00}\mathrm{kb}\,00 + \rho_{11}|b_\theta\rangle\langle b_\theta|.$$

The exact quantities are

$$\mathrm{boxed}\,\delta_{\mathrm{full}}(\theta) = \frac{\sin\theta}{2}, \qquad \varepsilon^R_{\mathrm{rec}}(\theta) = \frac{1-\cos\theta}{2}.$$

For *0 < θ < π/2*, the unitary *U_{θ}* is non-Clifford and its output on the stabilizer input $|+\rangle|0\rangle$ has positive bipartite non-local magic across *R: B*. At *θ = π/2*, the unitary is Clifford but the complement is still nonconstant.

Proof. Partial traces of the two columns in give the displayed channels. The residual pure states have overlap *cos θ*, hence trace distance *sin θ*. Lemma gives the lower bound *δ_full ≥ sin θ/2*.

Let

$$\sigma_\theta = \frac{1}{2}(\mathrm{kb}\,00 + |b_\theta\rangle\langle b_\theta|).$$

Each basis output has distance *sin θ/2* from *σ_{θ}*. For an arbitrary referenced input, the output difference is

$$\sum_{i=0}^{1} X_i \otimes (\sigma_i - \sigma_\theta),$$

where the diagonal reference blocks *X_i* are positive and *∑_i Tr X_i = 1*. The trace norm is therefore at most *sin θ*. This proves the matching half-diamond upper bound.

The radiation channel has

$$\Delta_c = \frac{1+c}{2}\mathrm{id} + \frac{1-c}{2}\mathrm{Ad}_Z.$$

The identity decoder has error *(1 − c)/2*, using an upper triangle bound and the orthogonal Bell outputs of *I* and *Z*. For any decoder of error *ε*, the ideal outputs of $|+\rangle$ and $|-\rangle$ have distance one, while their channel outputs have distance *c*. Contraction and the triangle inequality give *1 ≤ 2ε + c*. This proves the recovery formula.

For input $|+\rangle$, the complete output is

$$|\Psi_\theta\rangle = \frac{|0\rangle|0\rangle + |1\rangle|b_\theta\rangle}{\sqrt{2}}.$$

Its radiation reduction has eigenvalues *(1 + cos θ)/2* and *(1 − cos θ)/2*. For *0 < θ < π/2*, these are nonzero and unequal. Lemma shows that the state is not locally equivalent to a stabilizer state. In particular, *U_{θ}* cannot be Clifford, because it maps a stabilizer input to a nonstabilizer output.

At *θ = π/2*, *R_y(π) = XZ*, and

$$U_{\pi/2} = \mathrm{CNOT}\,\mathrm{CZ},$$

where *CZ* acts first. This is Clifford. On the allocated $|0\rangle$ residual, its code isometry is the copying isometry of Proposition .

par *Non-claim.* The formulas belong to this family; no exact gate synthesis of *U_{θ}* is asserted. par

This is the properly completed ``responding non-Clifford'' fragment: a specified non-Clifford coupling and a direct residual witness, rather than an inference from gate classification alone. The baseline *θ = 0* is nonresponding, and the response is proved on the same code, preparation, and partition.

For small *θ*,

$$\delta_{\mathrm{full}}(\theta) = \frac{\theta}{2} + O(\theta^3), \qquad \varepsilon^R_{\mathrm{rec}}(\theta) = \frac{\theta^2}{4} + O(\theta^4).$$

The exact relation is

$$\varepsilon^R_{\mathrm{rec}} = \frac{1 - \sqrt{1 - 4\delta^2_{\mathrm{full}}}}{2}.$$

This relation belongs to this family, not to arbitrary encodings.

Magic on $|\Psi_{\theta}\rangle$ vanishes at both endpoints, while leakage is zero only at the first endpoint. Thus even in this entangling family, leakage is not a faithful magic measure.

A general sufficient response condition

Theorem. Let *V_0: C → RB* have constant complement, let *K = K^{†}* be bounded, and define

$$V_\lambda = e^{-i\lambda K} V_0.$$

Fix code states *ρ_0*, *ρ_1*, put *X = ρ_0 − ρ_1*, and choose a bounded Hermitian residual observable *O* with *‖O‖ ≤ 1*. Suppose

$$m = \mathrm{Tr}\left[O\left(-i\mathrm{Tr}_R[K, V_0 X V_0^\dagger]\right)\right] \neq 0.$$

Then

$$\delta_{\mathrm{full}}(V_\lambda) \geq \max\left\{0, \frac{|m|\,|\lambda|}{4} - \|K\|^2\lambda^2\right\}.$$

In particular, the complement is nonconstant for

$$0 < |\lambda| < \frac{|m|}{4\|K\|^2}.$$

Proof. The derivative of the conjugated trace-class operator is *−i[K, V_0XV_0^{†}]*. The double commutator has trace norm at most *4‖K‖^2‖X‖_1*. Taylor's integral remainder is therefore bounded by *2λ^2‖K‖^2‖X‖_1 ≤ 4λ^2‖K‖^2*.

The baseline residual image of *X* is zero by constancy. Hence

$$|\mathrm{Tr}\,(O\mathscr{C}_\lambda(X))| \geq |m|\,|\lambda| - 4\lambda^2\|K\|^2.$$

Apply . The assumption $m \neq 0$ entails $\|K\| > 0$.

par *Non-claim.* The witness is sufficient only; Theorem shows that it is not necessary. par

The relevant additional premise is a nonzero residual matrix element of the coupling. A label such as ``interacting,'' ``non-Gaussian,'' or ``non-Clifford'' is not that premise. A vanishing first-order witness also does not rule out higher-order response.

Readout, forgetting, and the complete apparatus

The external residual program

For a finite encoded residual, the observer may explicitly add

$$P_{\mathrm{forgetB}} : Q_R \otimes Q_B \to Q_R$$

with body

x := Enter(B);?dropBit(x);?returnQ(R);

This is an external operation, not a conclusion about autonomous evolution.

Theorem. For every joint state, including an external reference,

$$\sum_x (\langle x|_B \otimes I)\rho(|x\rangle_B \otimes I) = \mathrm{Tr}_B\,\rho.$$

Consequently,

$$P_{\mathrm{forgetB}}\mathscr{V} = \mathscr{N},$$

with exactly the pre-existing radiation recovery error.

More generally, a trace-preserving local operation on B, with all its outputs ignored, leaves the untouched reference and R unchanged.

Proof. Partial trace is the sum of the diagonal blocks in any orthonormal basis. For a general local channel, its Heisenberg adjoint is unital. It therefore leaves every observable $A \otimes I_B$ on the untouched factors unchanged.

par *Non-claim.* This is operational forgetting at this interface, not permanent physical deletion. par

Thus the phrase ``empty the bin'' is only operational forgetting at this interface. It does not prove annihilation of the physical residual or its apparatus.

For an infinite residual, partial trace remains well defined. The theorem does not provide a finite program that measures infinitely many modes.

Proposition. For the instrument , define

$$W_{\mathrm{read}}|\psi\rangle = \sum_x |x\rangle_K|x\rangle_E M_x|\psi\rangle.$$

This is an isometry. Tracing E gives the observable destructive readout instrument. The complete output preserves the input globally.

Proof. Completeness of the M_x gives $W_{read}^{\dagger}W_{read} = I$. Tracing the orthogonal E-labels removes cross-outcome terms. Apply Theorem .

par *Non-claim.* The dilation models the apparatus; it is not a claim about a specific laboratory device. par

For a fully measured qubit,

$$a|0\rangle + b|1\rangle \longmapsto a|0\rangle_K|0\rangle_E + b|1\rangle_K|1\rangle_E.$$

The phase is absent from each classical marginal but retained jointly. The carrier K in this coherent dilation is a quantum system; its observable *Bit* interface arises after E is omitted.

This complete apparatus ledger is the distinction emphasized in [CheungHawk04B,CheungEm03C]. Operational discard may move a record carrier to an inaccessible environment in a unitary realization; it does not remove that environment from the complete preservation claim.

Theorem. If the only retained output of a complete process is classical, every subsequent deterministic reconstruction of C has error at least

$$1 - d^{-1}.$$

The same bound holds for every measure-and-prepare channel followed by a decoder. Computational readout followed by basis-state preparation attains it.

Proof. A finite classical-output channel has the form

$$X\mathrm{longmapsto}\sum_x \mathrm{Tr}\,(F_x X)|x\rangle\langle x|.$$

A decoder prepares one state for each outcome, giving a measure-and-prepare channel. Its Bell-input output is separable.

For a normalized product vector $u \otimes v$,

$$|\langle \Phi_d, u \otimes v\rangle|^2 \le 1/d.$$

Convexity gives the same Bell-overlap upper bound for separable states. Testing the Bell projector gives error at least $1 - 1/d$.

Computational dephasing is

$$\Delta_d = \frac{1}{d}\sum_{k=0}^{d-1} \mathrm{Ad}_{Z^k}.$$

The d Bell outputs are orthogonal, yielding distance $1 - 1/d$; the triangle inequality gives the matching upper bound.

par *Non-claim.* Supplied quantum side information changes the hypothesis. par

The entanglement-breaking characterization is established in [HorodeckiShorRuskai]. The quantitative statement is proved locally. A surviving quantum memory or supplied quantum side information changes the hypothesis.

Residual measurement outcomes supplied to the observer may assist quantum recovery. For example,

$$V|\psi\rangle = \frac{|\psi\rangle_R|0\rangle_B + Z|\psi\rangle_R|1\rangle_B}{\sqrt{2}}$$

has dephasing radiation. Measuring B gives a uniform bit x; applying $Z\hat{}x$ to R recovers exactly. Dropping the bit without using it leaves dephasing. The bit alone contains no diary. This is an access change, not a record-only quantum decoder.

The four-Majorana control is not a magic certificate

The independently specified model

Take four Majoranas with

$$\{\chi_i, \chi_j\} = 2\delta_{ij} I, \qquad S_j = \mathrm{i}\chi_{2j-1}\chi_{2j},$$

and freeze

$$H_S = J(I - S_1 S_2), \qquad A = \frac{S_1 + S_2}{2}, \qquad J > 0.$$

For $0 < \mu < \Lambda < \infty$, let

$$\mathfrak{h} = L^2([\mu, \Lambda], \mathrm{d}\omega), \qquad (h_B f)(\omega) = \omega f(\omega),$$

$$H_B = \mathrm{d}\Gamma(h_B), \qquad f(\omega) = \frac{1}{\sqrt{2\omega}}, \qquad H = H_S + H_B + gA \otimes \Phi(f).$$

The Hamiltonian is specified independently of Enter.

The field estimates

$$\|a(f)\psi\| \le \|h_B^{-1/2} f\| \|H_B^{1/2}\psi\|,$$

$$\|a^\dagger(f)\psi\|^2 = \|a(f)\psi\|^2 + \|f\|^2\|\psi\|^2$$

give arbitrarily small relative bound of the interaction with respect to *H_B*. Since *H_S* and *A* are bounded, Kato–Rellich gives self-adjointness on *D(H_B)* [Kato]. The finite cutoffs make the displayed norms finite.

Prepare the source in $|11\rangle_S$ and the bath in vacuum. Swap an even-parity two-mode diary into the source:

$$|0\rangle_C \mapsto |00\rangle_S, \qquad |1\rangle_C \mapsto |11\rangle_S.$$

The displaced carrier is the fixed state $|11\rangle_D$. The logical source states *s_0, s_1* have zero source energy and coupling eigenvalues *a_0 = −1, a_1 = 1*.

For a declared packet projection *P_{r}*, set

$$\eta_i(t) = g a_i \frac{e^{-i\omega t} - 1}{\omega} f(\omega),$$

$$\eta_{i,R} = P_r \eta_i, \qquad \eta_{i,U} = (I - P_r)\eta_i,$$

$$c_R = \exp\left[-\frac{1}{2}\|\eta_{0,R} - \eta_{1,R}\|^2\right], \qquad z_R = \sqrt{1 - c_R^2}.$$

Radiation is the collected bath; the source, carrier, and uncollected bath form the complete residual.

Theorem. The complete code vectors, up to a common phase, are

$$V_t|i\rangle = |\eta_{i,R}\rangle_R \otimes |s_i\rangle_S |11\rangle_D |\eta_{i,U}\rangle.$$

They give

$$\mathcal{N}_t(\rho) = \rho_{00}|\eta_{0,R}\rangle\langle\eta_{0,R}| + \rho_{11}|\eta_{1,R}\rangle\langle\eta_{1,R}|,$$

and the exact formulas

$$\text{boxed}\, \varepsilon_{\text{rec}}^R(t) = \frac{3 - z_R}{4}, \qquad \varepsilon_{\text{rec}}^B(t) = \frac{1 - c_R}{2}, \qquad \delta_{\text{full}}(t) = \max\left\{\frac{1}{2}, \frac{1 + 2c_R}{4}\right\}.$$

In particular, *ε_rec^R ≥ 1/2* and *δ_full ≥ 1/2*. The complete output is exactly reversible.

Proof. On source eigenvalue *a_i*, let *q_i = g a_i h_B^{−1} f*. The Weyl identity gives

$$H_B + g a_i \Phi(f) = \mathcal{W}(q_i)^\dagger H_B \mathcal{W}(q_i) - g^2 a_i^2 \|h_B^{-1/2} f\|^2 I,$$

where $\mathcal{W}(q) = e^{a^\dagger(q) - a(q)}$. The vacuum therefore evolves to the coherent amplitude $(e^{-ith_B} - I)q_i = \eta_i(t)$. The phase is common for *a_i = ±1*. Orthogonal packet decomposition factors the coherent state. Tracing the orthogonal source labels gives the radiation channel.

Its two pure output states have trace distance *z_R*. Their optimal equal-prior discrimination probability is *(1 + z_R)/2*, obtained by the positive part of their difference. For any radiation decoder, its Bell overlap is

$$F_e = \frac{1}{4}[\langle 0|\mathcal{D}(\sigma_0)|0\rangle + \langle 1|\mathcal{D}(\sigma_1)|1\rangle] \le \frac{1 + z_R}{4}.$$

The two diagonal decoder effects constitute a discrimination POVM. The Bell-projector test gives error at least *(3 − z_R)/4*.

Bath parity interchanges the two states, so an optimal discrimination POVM can be symmetrized to have equal conditional errors. Measurement followed by preparation of the inferred basis state gives

$$z_R \Delta_Z + (1 - z_R)\mathcal{T}_{I_2/2}.$$

Its Pauli probabilities are

$$p_I = p_Z = \frac{1 + z_R}{4}, \qquad p_X = p_Y = \frac{1 - z_R}{4}.$$

Orthogonal Pauli Bell outputs and the triangle inequality give half-diamond error *1 − p_I*, proving the radiation formula.

Define the isometry

$$L_t|i\rangle = |s_i\rangle_S|11\rangle_D|\eta_{i,U}\rangle.$$

Then

$$\mathscr{C}_t = L_t\Delta_{c_R}(\,\cdot\,)L_t^\dagger.$$

Retracting the range of *L_t* gives the dephasing recovery error *(1 − c_R)/2*, by the same upper bound and $|+\rangle$, $|-\rangle$ lower bound used in Theorem .

Isometric embedding and CPTP retraction preserve leakage. For *Δ_c*, Pauli covariance and convexity show that twirling any candidate replacement state to *I_2/2* cannot increase its distance. Relative to that replacement, the Pauli coefficients, in order *I*, *Z*, *X*, *Y*, are

$$\frac{1+2c}{4},\quad \frac{1-2c}{4},\quad -\frac{1}{4},\quad -\frac{1}{4}.$$

The half-diamond norm is half the sum of their absolute values: a Bell input gives the lower bound and the triangle inequality gives the upper bound. Hence

$$\delta_{\text{full}} = \frac{3+2c+|1-2c|}{8} = \max\left\{\frac{1}{2}, \frac{1+2c}{4}\right\}.$$

The complete columns are orthonormal, so global recovery follows from Theorem .

par *Non-claim.* Not a magic, scrambling, or gravitational certificate. par

The radiation obstruction is [CheungHawk04A]; the residual formulas are the refinements in [CheungHawk04B]. They are rederived here.

The baseline prevents a magic-origin inference

At *g = 0*, or at *t = 0*, collected radiation is vacuum and *c_R = 1*. Therefore

$$\varepsilon^R_{\text{rec}} = \frac{3}{4}, \qquad \varepsilon^B_{\text{rec}} = 0, \qquad \delta_{\text{full}} = \frac{3}{4}.$$

Residual non-constancy is already maximal for a qubit at the initial interface: the diary is retained in the inaccessible source. Nonzero coupling is not the cause of that pre-existing non-constancy.

As collected coherent states become more distinguishable, the radiation error decreases and residual leakage can decrease. Thus this control cannot establish that added non-Clifford resources force added leakage.

No fixed qubit Pauli dictionary for the entire bosonic dynamics has been declared, and no magic measure has been evaluated for it. The control is commuting and uses no scrambling or out-of-time-order estimate. It is a recovery obstruction, not a certificate of non-local magic or geometric response.

If all bath modes are accessible,

$$c_R(t) = \exp\left[-2g^2\int_\mu^\Lambda \frac{1-\cos(\omega t)}{\omega^3}\mathrm{d}\omega\right].$$

For a restricted packet space, a nonzero coupling need not produce nonzero collected displacement at every chosen time.

The source energy remains zero, while

$$E_B(t) = g^2\int_\mu^\Lambda \frac{1-\cos(\omega t)}{\omega^2}\mathrm{d}\omega, \qquad E_I(t) = -E_B(t).$$

This follows from coherent bath energy and total-energy conservation. It is dressing of an initially undressed preparation, not a gravitational evaporation law.

Finite certificates and what they actually establish

Choi and complementary-channel comparisons

Lemma. For channels of input dimension *d*,

$$d_\diamond(\Phi,\Psi) \le \min\left\{1, \frac{d}{2}\|J_{\text{n}}(\Phi) - J_{\text{n}}(\Psi)\|_1\right\}.$$

Proof. A normalized pure referenced input has the form *(L ⊗ I)|Φ_d⟩*, with *Tr (L^{†}L) = d* and *‖L‖^2 ≤ d*. Multiplying the normalized Choi difference on both sides by *L* bounds its trace norm by *d* times the Choi trace norm. Convexity handles mixed inputs.

par *Non-claim.* A conversion inequality only. par

For finite complementary channels, the KSW information–disturbance theorem [KSW] implies, in the present half-diamond convention,

$$\frac{\delta_{\mathrm{full}}^2}{2} \le \varepsilon_{\mathrm{rec}}^R \le \sqrt{2\delta_{\mathrm{full}}}.$$

Indeed, in full-diamond normalization the recovery-to-leakage direction gives

$$\inf_{\sigma} \| \mathscr{C} - \mathscr{T}_\sigma \|_\diamond \le 2\sqrt{\inf_{\mathscr{D}} \| \mathscr{D}\mathscr{N} - \mathrm{id} \|_\diamond},$$

and the opposite direction gives the analogous bound with recovery and leakage exchanged. Division by two yields . This is a channel conversion, not a magic theorem or a dynamical estimate.

Theorem. Suppose a finite prepared proxy is embedded access-preservingly in a common output space and satisfies

$$d_\diamond(\mathscr{V}, \widehat{\mathscr{V}}) \le b.$$

Then

$$\left|\delta_{\mathrm{full}} - \widehat{\delta}_{\mathrm{full}}\right| \le b, \qquad \left|\varepsilon_{\mathrm{rec}}^R - \widehat{\varepsilon}_{\mathrm{rec}}^R\right| \le b.$$

If a validated proxy pair witness gives

$$\widehat{x} = d_{\mathrm{tr}}(\widehat{\mathscr{C}}(\rho_0), \widehat{\mathscr{C}}(\rho_1)),$$

then

$$\delta_{\mathrm{full}} \ge \max\{0, \widehat{x}/2 - b\}.$$

For a specified proxy replacement state, put

$$e = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{C}}) - \frac{I_Q}{d} \otimes \widehat{\sigma} \right\|_1.$$

Then

$$\delta_{\mathrm{full}} \le \min\{a_d, de + b\},$$

and a finite proxy decoder has a full extension satisfying

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{a_d, \sqrt{2de} + b\}.$$

The last bound means that one may choose the better of the extended proxy decoder and the constant decoder.

Proof. Partial trace gives allowance *b* for both reduced channels. A local isometric embedding has a CPTP retraction

$$Y \mathrm{longmapsto} J^\dagger Y J + \mathrm{Tr}\left((I - JJ^\dagger)Y\right)\tau.$$

Embedding and retracting decoders and replacement states preserve the tasks. Lemma therefore gives stability.

The pair-witness lower bound follows from Lemma and leakage stability. The Choi estimate gives proxy leakage at most *de*. Equation supplies a finite decoder of error at most *√(2de)*. Extend it off the proxy support by fixed-state preparation. Contraction adds *b*. Lemma supplies the cap.

par *Non-claim.* Finite-channel certificates concern response and recovery, not magic. par

A specified decoder can instead be certified by its normalized logical Choi error *r*, giving allowance *dr + b*. CPTP feasibility must be validated, including positivity and trace preservation on the entire finite input space. An optimization objective alone is not such a certificate.

Finite-channel certificates concern response and recovery. They do not certify magic unless a separate fixed-frame resource calculation is supplied.

A finite-time microscopic allowance

For completeness, the regulator used by the source–scalar architecture can be stated independently of the magic claims. Let the source be finite dimensional, let H_S be bounded, and let

$$H = H_S + \mathrm{d}\Gamma(h_B) + \sum_a g_a A_a \otimes \Phi(f_a),$$

with self-adjoint bounded A_a, vacuum bath preparation, and $\mu I \le h_B \le \Lambda I$. Define

$$G = \sum_a |g_a| \|A_a\| \|f_a\|, \qquad Z_m(x) = \sum_{\ell = m+1}^{\infty} \frac{x^\ell}{\sqrt{\ell!}}.$$

Proposition. For finitely many declared packets, occupation cutoff m, frequency bins of width at most Δ, and $0 \le t \le T$, one can construct a finite active proxy with

$$d_\diamond(\mathscr{V}_t, \widehat{\mathscr{V}}_t) \le \min\left\{1, 2Z_m(2GT) + \frac{Tm\Delta}{2}\right\}.$$

The embedding preserves the accessible–inaccessible partition.

Proof. In interaction picture, on occupation at most k,

$$\|V_I(s)\psi\| \le 2G\sqrt{k+1}\,\|\psi\|.$$

An ℓ-fold product starting from vacuum has norm at most $(2G)^\ell/\sqrt{\ell!}$. Ordered integration gives Dyson-term bound $(2Gt)^\ell/\sqrt{\ell!}$, uniformly over code–reference inputs. These series converge also after multiplication by any fixed occupation power; since h_B is bounded, termwise differentiation on prepared vectors identifies the series with the self-adjoint evolution.

Compression to occupation at most m leaves all terms through order m unchanged. Higher terms in either series obey the same bound, giving prepared-isometry difference at most $2Z_m(2GT)$.

Replace frequency by the midpoint of each bin. The one-particle operator difference has norm at most $\Delta/2$; on occupation at most m, its second quantization has norm at most $m\Delta/2$. Duhamel's formula adds $Tm\Delta/2$.

The span of the bin-restricted form factors and packet functions is finite dimensional and invariant under the midpoint operator. Modes orthogonal to it stay in vacuum. Its total-occupation cutoff is therefore a finite active proxy. The total cutoff embeds in separate collected and uncollected cutoff Fock spaces; it is not equal to their full tensor product. The source and displaced carrier remain in the residual.

Lemma converts prepared-isometry error to the complete-channel allowance.

par *Non-claim.* The infrared and ultraviolet cutoffs are not removed. par

This is the finite-time comparison discipline of [CheungHawk02E,CheungHawk04B]. It does not remove the infrared or ultraviolet cutoffs. Separate finite qubit dictionaries preserve the channel task but do not construct a local physical receiver or transport a magic classification automatically.

Compatible half-diamond preparation, collection, and implementation allowances telescope under CPTP composition [CheungEn02B,CheungEn02C]. The flagged-cutoff equalities in [CheungEn02E] retain their commuting-state hypotheses. A discarded probability is not a generic coherent-channel error.

What would be needed for a geometric interpretation

A declared geometric subsystem is additional data

Suppose a physical model identifies

$$B = G \otimes E$$

and specifies a collection $\mathcal{O}_G$ of bounded Hermitian observables intended to represent geometric quantities. Define

$$\mathscr{C}_G = \mathrm{Tr}_E\, \mathscr{C}.$$

This declaration is stronger than merely naming B ``geometry."

Proposition. If some $O_G \in \mathcal{O}_G$, with $\|O\|_G \le 1$, and some code states satisfy

$$\mathrm{Tr}\,(O_G \mathscr{C}_G(\rho_0 - \rho_1)) \neq 0,$$

then the complete residual is nonconstant and exact whole-code recovery from R is impossible.

The converse is false.

Proof. Lift the observable to $O_G \otimes I_E$ and apply Lemma and Theorem .

For the converse, take

$$V|\psi\rangle = |0\rangle_R|\mathrm{vac}\rangle_G|\psi\rangle_E.$$

The complete residual recovers the diary and is nonconstant, but $\mathscr{C}_G = \mathscr{T}_{|vac\rangle\langle vac|}$. Every declared geometric observable is input independent.

par *Non-claim.* No observable is identified as geometric by this proposition; Theorem gives the classification. par

This counterexample also shows why vanishing source energy next to an inaccessible diary memory does not imply residual independence [CheungHawk02E]. Choose $H_G|vac\rangle = 0$; its energy is always zero despite complete residual response.

Entropic rigidity is not a universal geometry theorem

In finite dimension, the exact normal form gives

$$S(\mathcal{N}(\rho)) = S(\rho) + S(\sigma_K).$$

The additive syndrome entropy is independent of the logical input. This follows directly from tensor-product entropy and isometric invariance.

Conversely, a selected entropy does not locate the code. For

$$V_{\mathrm{good}}\psi = \psi_R|0\rangle_B, \qquad V_{\mathrm{bad}}\psi = |0\rangle_R\psi_B,$$

both radiation entropies vanish on every pure code input. Their radiation recovery errors are 0 and a_d. An entropy identity alone is not the complete complementary-channel criterion.

Cao's area-operator result concerns stabilizer codes with the specified complementary operator-algebra structure, not arbitrary Clifford channels [Cao2024]. The skewed-code analysis of Cao and collaborators relates leading proto-area entropy corrections to a specially defined perturbative tripartite Choi magic under its subsystem and perturbation assumptions [CaoEtAl2026]. Neither statement equates generic complete residual non-constancy with geometric backreaction.

The syndrome counterexample in Theorem has bipartite magic but no logical response. It therefore does not contradict a theorem whose resource is a more restricted matter–geometry tripartite coupling. It demonstrates why those restrictions matter.

The corrected conditional bridge

Corollary. Fix a physical code, complete preparation, unitary deformation, and access partition. Suppose:

• a residual subsystem or observable is justified as geometric by an independent same-model dictionary;

• the deformation has a nonzero response witness in that observable, such as the coefficient in Theorem ;

• any claimed magic resource is independently defined and certified in the same carrier convention. Then the certified observable responds to the declared code inputs, the complete complement is nonconstant, and exact whole-code radiation recovery is obstructed.

The magic certificate does not replace the response witness or the geometric dictionary.

Proof. The response witness and the dictionary identify the intended observable variation. Proposition gives nonconstancy and the exact-recovery obstruction.

par *Non-claim.* No metric equation, stress–energy identification, backreaction law, or physical endpoint is supplied; see Theorem for which hypothesis buys which conclusion. par

A metric equation, stress–energy identification, backreaction law, and physical endpoint require further same-model constructions. None is supplied by this corollary.

Likewise, a continuum claim about vacuum magic is not a finite carrier leakage theorem. For example, the thermodynamic-limit magic analysis of [Moosa2026] concerns a different algebraic setting. Even if a particular vacuum resource theorem is applicable, a bridge to this task still needs a preparation channel, a finite dictionary, a complete residual, and a response estimate. No intrinsic-magic claim for a Lorentz-invariant vacuum is used as a premise here.

Endpoint continuity and recurrence

Access-preserving limits

For time-dependent partitions *R_tB_t*, let

$$J_{R,t} : R_t \to R_\infty, \qquad J_{B,t} : B_t \to B_\infty$$

be local isometries, and put

$$\overline{V}_t = (J_{R,t} \otimes J_{B,t}) V_t.$$

These do not move an inaccessible diary factor into radiation.

Theorem. If $\overline{V}_t|i\rangle \to V_\infty|i\rangle$ on a code basis, then V_∞ is an isometry and

$$a_t = \left\| \overline{V}_t - V_\infty \right\| \to 0.$$

Complete and reduced channels converge in half-diamond distance with allowance a_t, and

$$|\delta_{\mathrm{full}}(t) - \delta_{\mathrm{full}}(\infty)| \le a_t.$$

Thus a proved limit $\delta_{full}(t) \to 0$ gives exact endpoint recovery from radiation.

Proof. Finite-dimensionality of the code gives

$$\left\| \overline{V}_t - V_\infty \right\|^2 \le \sum_i \left\| (\overline{V}_t - V_\infty)|i\rangle \right\|^2 \to 0.$$

Inner products pass to the limit. Lemma gives channel convergence. Local embeddings preserve leakage by the CPTP retraction used in Theorem . Zero endpoint leakage implies exact constancy by Lemma ; apply Theorem .

par *Non-claim.* No gravitational endpoint and no Page curve is constructed (Theorem). par

This is the access-preserving endpoint interface in [CheungHawk04A,CheungHawk04B]. A unitary swapping R and B changes the recovery task and is not an admissible local identification.

If the endpoint residual is fixed pure, endpoint radiation is an isometric encoding. A mixed fixed residual gives the syndrome normal form. An infinite endpoint syndrome need not admit a finite decoder program.

Fixed finite proxies return

Proposition. For a fixed finite-dimensional autonomous Hamiltonian, there are arbitrarily late times at which its prepared complete channel is arbitrarily close to its initial channel.

If initially accessible radiation is fixed vacuum and the complete prepared map is an isometry, then

$$\limsup_{t \to \infty} \varepsilon^R_{\mathrm{rec}}(t) = a_d, \qquad \limsup_{t \to \infty} \delta_{\mathrm{full}}(t) = a_d.$$

These statements do not assert positive leakage at every time.

Proof. The finitely many spectral phases evolve in a compact torus. Simultaneous recurrence gives arbitrarily late unitary returns to the identity; periodic cases use multiples of a period. Lemma gives prepared-channel returns.

At time zero, radiation is constant, so its recovery error is a_d. The complete isometry with pure fixed radiation has an isometric residual encoding, whose leakage is a_d by Lemma and isometric invariance. Distance stability gives both values along the recurrence sequence. The universal upper bounds in Lemma give the limsup equalities.

par *Non-claim.* Neither positive leakage at every time nor monotone leakage is asserted. par

This is the fixed-proxy obstruction of [CheungHawk02E], refined using the complete-channel distances. It gives

recurrent nonresponse failure when the initial task has that failure. It neither proves monotone leakage nor identifies its origin as magic.

A finite acyclic program denoting *U(t)* terminates at each fixed *t*. That syntactic fact proves no asymptotic endpoint, no permanent release, and no disappearance of *B*. A finite-time regulator must precede a controlled outgoing limit.

Checkable criteria and typed refusals (new in v2)

This section adds eight theorems. Each is local to the finite carrier and is proved here. Cited sheets of the corpus are source records. Citations identify source records only; no proof transfer. No quantum coefficient is admitted.

Throughout, $V: C \to R \otimes B$ is an isometry, $C \simeq \mathbb{C}^d$, $\mathscr{C} = Tr_R \mathscr{V}$, $\mathscr{N} = Tr_B \mathscr{V}$, and δ_{full}, ε_{rec}^R are and . A *declared geometric factor* is a unitary identification $B \simeq G \otimes E$ together with a set $\mathcal{O}_G$ of bounded Hermitian observables on G; its *witnesses* are the numbers

$$w(O_G; \rho_0, \rho_1) = \mathrm{Tr}\,(O_G\, \mathscr{C}_G(\rho_0 - \rho_1)), \qquad \mathscr{C}_G = \mathrm{Tr}_E\, \mathscr{C}.$$

The trivial readings are excluded: a scalar O_G or $\dim G = 1$ has zero witness for every V and is not counted.

Residual response never forces a geometric factor

Theorem. Let $V: C \to R \otimes B$ have nonconstant complete complement.

• *Criterion.* For a declared factor $B \simeq G \otimes E$ and a bounded Hermitian O_G, all witnesses of O_G vanish if and only if

$$V^\dagger (I_R \otimes O_G \otimes I_E) V \in \mathbb{R}\, I_C.$$

All witnesses of all observables on G vanish, that is $\mathscr{C}_G$ is constant, if and only if one CPTP decoder on $R \otimes E$ recovers the entire code exactly.

• *No forced factor.* For every Hilbert space G with $\dim G \geq 2$ and every unit vector $g_0 \in G$, the carrier

$$V^G|\psi\rangle = V|\psi\rangle \otimes |g_0\rangle_G, \qquad B^G = B \otimes G, \quad E := B,$$

has the same radiation channel $\mathscr{N}$, the same ε_{rec}^R, and $\delta_{full}(V^G) = \delta_{full}(V) > 0$. Its complete complement still responds, while $\mathscr{C}_G = \mathscr{T}_{|g_0\rangle\langle g_0|}$: every Hermitian O_G, scalar or not, has zero witness.

• *Exact scope on a fixed residual.* Let $\dim B = n < \infty$.

• A nontrivial blind factor of B itself ($B \simeq G \otimes E$, $\dim G \geq 2$, $\mathscr{C}_G$ constant) exists if and only if some such identification makes the code exactly recoverable from $R \otimes E$. None exists if n is prime, and none exists for $V_{bad}|\psi\rangle = |0\rangle_R|\psi\rangle_B$ with $n = d$, for any $d \geq 2$.

• If $n \geq d + 1$, there is a non-scalar Hermitian O on B whose witnesses all vanish. For $n \leq d$ this can fail: for V_{bad} only scalar observables have vanishing witnesses.

• *Converse.* If some declared O_G has a nonzero witness, exact whole-code recovery from R is impossible.

Proof. (a) By duality, $w(O_G; \rho_0, \rho_1) = Tr\,(V^\dagger (I_R \otimes O_G \otimes I_E) V(\rho_0 - \rho_1))$. Differences of states span the traceless Hermitian operators on C, and a Hermitian operator that is orthogonal to all of them is a real multiple of I_C. The second statement is Theorem , (i)⇔(ii), applied to the same isometry with accessible output $R \otimes E$ and residual G. No new recovery theorem is used.

(b) $\mathscr{V}^G(X) = \mathscr{V}(X) \otimes |g_0\rangle\langle g_0|$. Tracing B and G gives $\mathscr{N}$, so the recovery task on R is unchanged. The new complement is $\mathscr{C} \otimes |g_0\rangle\langle g_0|$. The map $Y \mapsto Y \otimes |g_0\rangle\langle g_0|$ is an isometric CPTP embedding with CPTP retraction Tr_G, so replacement states, distances, and δ_{full} are preserved, as in Theorem . Tracing $E = B$ gives $\mathscr{C}_G(X) = Tr\,(X)|g_0\rangle\langle g_0|$.

(c1) The first sentence restates (a) for B fixed. If n is prime, $\dim G \in \{1, n\}$; $\dim G = n$ forces $\dim E = 1$ and $\mathscr{C}_G \simeq \mathscr{C}$, which is nonconstant. For V_{bad}, $\mathscr{C} = id$ up to the identification, so for every $B \simeq G \otimes E$ with $\dim G \geq 2$, $\mathscr{C}_G = Tr_E \circ Ad_W$ for a unitary W, which is surjective onto the states of G and not constant.

(c2) The compression $\Gamma(O) = V^\dagger (I_R \otimes O) V - d^{-1} Tr\,(V^\dagger (I_R \otimes O) V) I_C$ is a real-linear map from the n^2-dimensional space of Hermitian operators on B into the $(d^2 - 1)$-dimensional space of traceless Hermitian

operators on C. Its kernel contains I_B and has dimension at least $n^2 - d^2 + 1$, which is at least 2 exactly when $n > d$. A kernel element that is not a multiple of I_B has zero witnesses by (a). For V_{bad}, $\Gamma(O) = O - d^{-1}\mathrm{Tr}(O)I$, whose kernel is $\mathbb{R}I$.

(d) A nonzero witness makes $V^{\dagger}(I_R \otimes O_G \otimes I_E)V$ non-scalar. Theorem , (v)⇒(i) fails. This is the first half of Proposition .

par *Non-claim.* Theorem does not say that any factor is geometric, and it does not say that the plan's literal sentence holds for a fixed residual: by (c), for V_{bad} or prime $\dim B$ no nontrivial blind factor of B exists. The universal form (b) adjoins a fixed factor. Discard is not deletion: CheungHawk04B. En04-B's hosted thm:blind CheungEn04B is not imported. par

Remark. The plan states that for every isometry with nonconstant complement there is a factorization $B = G \otimes E$ with nonempty $\mathcal{O}_G$ of vanishing witnesses. Read literally this is true only trivially ($\dim G = 1$ or scalar observables). With $\dim G \geq 2$ and a non-scalar observable required, it holds after adjoining a fixed factor (b), on B itself exactly under the recovery condition (c1), and for single non-scalar observables whenever $\dim B > d$ (c2). It fails for V_{bad} and for prime $\dim B$ in the full-factor form.

The conditional bridge: which hypothesis buys which conclusion

Split the conclusion of Corollary into three typed parts:

(C1) the complete complement is nonconstant and exact whole-code radiation recovery is obstructed;

(C2) a dictionary-declared geometric observable responds to the declared code inputs;

(C3) that response is accompanied by an independently certified non-Clifford resource in the same carrier convention. (C3) is the content of the original slogan; (C1) and (C2) are what Corollary prints.

Theorem. Let (i) be the same-model dictionary, (ii) the nonzero witness in the declared observable, and (iii) the resource certificate of Corollary .

• A nonzero witness in any residual observable gives (C1); (i) and (ii) give (C2); (i), (ii), and (iii) give (C3).

• *Drop (i).* On the single carrier $V_{\theta}^{G} = V_{\theta} \otimes |0\rangle_G$, $0 < \theta < \pi/2$, with V_{θ} from , the residual witness $\mathrm{Tr}(Z_E \mathscr{C}_E(kb\ 00 - kb\ 11)) = 2\sin^2\theta \neq 0$ and the certificate (iii) (U_{θ} non-Clifford, Theorem) both hold. The declaration ``$E$ is geometric'' gives (C2); the declaration ``G is geometric'' gives not-(C2). Without (i), (C2) is not determined.

• *Drop (ii).* With the declaration ``G is geometric'' on the same carrier, (i) and (iii) hold, every witness on G vanishes, and (C2) and (C3) fail while (C1) holds.

• *Drop (iii).* For the Clifford copier $V_{copy}|i\rangle = |i\rangle_R|i\rangle_B$ with the declaration ``B is geometric'', (i) and (ii) hold ($\mathrm{Tr}(Z_B \mathscr{C}(kb\ 00 - kb\ 11)) = 2$), so (C1) and (C2) hold, but (C3) fails: V_{copy} has a Clifford dilation and a stabilizer Choi state.

Proof. (a) Lemma and Theorem give (C1) from any nonzero residual witness. (C2) is the conjunction of (i) and (ii). (C3) adds (iii) by definition.

(b), (c) The residual columns of V_{θ} are $|0\rangle$ and $|b_{\theta}\rangle$, so $\mathrm{Tr}(Z(kb\ 00 - |b_{\theta}\rangle\langle b_{\theta}|)) = 1 - \cos 2\theta = 2\sin^2\theta$. By Theorem (b), $\mathscr{C}_G$ is constant. Theorem certifies that U_{θ} is non-Clifford for $0 < \theta < \pi/2$. The two declarations are made on one and the same carrier, so the carrier, its witness on E, and its resource certificate cannot select between them.

(d) $V_{copy} = \mathrm{CNOT}(\cdot \otimes |0\rangle)$. The residual columns are $|0\rangle$, $|1\rangle$. The Choi vector $(|000\rangle + |111\rangle)/\sqrt{2}$ is a stabilizer state. By Lemma no non-Clifford resource is present in the declared frame.

par *Non-claim.* Theorem supplies no dictionary and no resource certificate. OpenAI Families 265, 272, 276, and 280 OAI265,OAI272,OAI276,OAI280 are not this dictionary and not this certificate. The two declarations in (b) are hypothetical labels, not physical identifications. par

Remark. The plan asks that each of the three hypotheses of Corollary be necessary for its conclusion. For the printed conclusion, (C1) and (C2), hypothesis (iii) is not used (the corollary itself says that the certificate replaces neither the witness nor the dictionary), and (C1) needs only a residual witness. Each of (i), (ii), (iii) is necessary for (C3). This is the form proved.

Leakage is not a function of a declared magic label

The resource labels need a qubit frame. Let $d = 2^k$, and fix qubit frames on C, R, B; for other d a padding convention as in Section must be declared first. A *Clifford dilation* of V is a Clifford unitary U with $V|\psi\rangle = U(|\psi\rangle \otimes |0^m\rangle)$.

Lemma. If V has a Clifford dilation, V maps every pure stabilizer input to a pure stabilizer output, and its normalized Choi vector $(I_Q \otimes V)|\Phi_d\rangle$ is a stabilizer state. Consequently, if V maps one stabilizer input to a nonstabilizer vector, V has no Clifford dilation.

Proof. $|s\rangle \otimes |0^m\rangle$ is a stabilizer state; Clifford unitaries normalize the Pauli group and so map stabilizer groups to stabilizer groups. For the Choi vector, $|\Phi_d\rangle$ is a stabilizer state and $I_Q \otimes U$ is Clifford.

par *Non-claim.* Lemma is a one-way test; it is not a synthesis or compilation theorem. par

The three labels, made well defined for isometries:

• *Clifford flag*: $Cl(V) = 1$ if V has a Clifford dilation, and 0 otherwise.

• *T-count.* A T-count belongs to a circuit, not to a channel: $T^2 = S$ and $TT^\dagger = I$ give different counts for Clifford isometries. Two well-defined versions are used: the declared count $t(c)$ of a declared Clifford+T circuit c, and the minimal count $\tau(V) \in \{0, 1, 2, \ldots\} \cup \{\infty\}$ over all exact Clifford+T circuits with $|0\rangle$ ancillas. Here $\tau(V) = 0$ if and only if $Cl(V) = 1$.

• *Stabilizer-entropy bin*: $M_2(V) = -\log_2 (2^{-n}\sum_P \langle\Psi_V|P|\Psi_V\rangle^4)$ for the normalized Choi vector Ψ_V on n qubits, read through a declared partition of $[0, \infty)$ into intervals (bins). It is phase invariant and therefore a function of V.

Theorem. Fix $d = 2^k \geq 2$ and the output frames. Let the data be the code dimension, the output spaces and frames, and one declared label from (L1)–(L3). Then:

• *Flag.* Each fibre $Cl = 1$ and $Cl = 0$ contains carriers with $\delta_{full} = 0$ and $\delta_{full} = 1/2$.

• *T-count.* For every declared circuit count $t \geq 0$, and for each minimal count $\tau \in \{0, 1\}$, the fibre contains carriers with $\delta_{full} = 0$ and $\delta_{full} = 1/2$.

• *Stabilizer entropy.* For every $\mu \geq 0$ there are carriers with $M_2 = \mu$ exactly, the same output spaces, and $\delta_{full} = 0$ and $\delta_{full} = 1/2$. Hence every bin is such a fibre.

• *Indicator error.* On each of these fibres, every $[0, 1]$-valued report F of the indicator $1[\delta_{full} > 0]$ has worst-case absolute error at least $1/2$; the constant report $1/2$ attains it. The same holds in expectation for randomized $\{0, 1\}$ reports. For δ_{full} itself, the worst-case absolute error is at least $1/4$.

Proof. Work with $k = 1$; for $k > 1$ pass the extra $k - 1$ logical qubits to R unchanged. That adds Bell pairs to the Choi vector (M_2 is additive and zero on them), keeps (L1) and (L2), and does not change δ_{full}: precomposition with the partial trace is a contraction, and restriction to product inputs gives the reverse inequality.

For a phase gate $P(a) = \mathrm{diag}(1, e^{ia})$, set

$$V_a^0|\psi\rangle = P(a)|\psi\rangle_R|0\rangle_B, \qquad V_a^1 = (P(a)_R \otimes I_B)V_{\mathrm{copy}}.$$

Then $\mathscr{C}(V_a^0)$ is constant and $\mathscr{C}(V_a^1) = \Delta_Z$, so $\delta_{full} = 0$ and $\delta_{full} = 1/2$ by Proposition .

(a) $Cl = 1$: V_0^0 and $V_0^1 = V_{copy}$. $Cl = 0$: $V_T = V_{\pi/4}^0$ and $V_{\pi/4}^1$. On the stabilizer input $|+\rangle$ these give $T|+\rangle \otimes |0\rangle$ and $(|00\rangle + e^{i\pi/4}|11\rangle)/\sqrt{2}$, neither a stabilizer state (their Pauli spectra contain the values $\pm 2^{-1/2}$); apply Lemma .

(b) Declared count t: the diagonal circuit $D_t = \ldots T^\dagger T$ with t alternating factors, followed or not by $CNOT_{C \to B}$. Diagonal gates commute with the CNOT control, so the two complements are constant and Δ_Z. Minimal count: $\tau = 0$ is (a) with $Cl = 1$. For $\tau = 1$: $V_{\pi/4}^0$ and $V_{\pi/4}^1$ each have a circuit with one T, and $\tau \geq 1$ by (a).

(c) For $GHZ_n(a) = (|0^n\rangle + e^{ia}|1^n\rangle)/\sqrt{2}$, the nonzero Pauli expectations are 1 on the 2^{n-1} even Z-strings and $\cos(a - j\pi/2)$ on the strings with X or Y on every qubit and j factors Y. Hence

$$\sum_P \langle P\rangle^4 = 2^{n-1}(1+\cos^4 a+\sin^4 a), \qquad M_2 = -\log_2 \frac{1+\cos^4 a+\sin^4 a}{2},$$

independently of *n*. The Choi vector of *V_a^0* is *GHZ_2(a)* ⊗ |*0*⟩, and that of *V_a^1* is *GHZ_3(a)*. So the two carriers have equal *M_2*, which increases continuously from *0* at *a = 0* to *log_2 (4/3)* at *a = π/4*. Appending the fixed state *(T|+⟩)^{⊗j}* to *R* in both adds exactly *jlog_2 (4/3)* to *M_2* and leaves *δ_full* unchanged, by the embedding argument of Theorem (b) on the radiation side. The intervals *[j, j + 1]log_2 (4/3)* cover *[0, ∞)*.

(d) This is the two-point fibre argument: if *F* is constant on a fibre containing indicator values *0* and *1*, then *max {|F − 0|, |F − 1|} ≥ 1/2*. The same argument applied to the *δ_full* values *0* and *1/2* gives *1/4*.

par *Non-claim.* Theorem evaluates no magic monotone beyond the declared labels, asserts no lower bound on minimal T-count above *1*, and identifies no label with a geometric resource. Big01-A thm:fibre CheungBig01A is the pattern and is not applied to a host. Big04-A-coeff thm:C-adaptive CheungBig04Acoeff is the parallel statement for a different slot (a coefficient κ); neither fills this one. par

Remark. PC-3: the labels need a qubit frame, so the theorem is stated for *d = 2^k*; other *d* require a declared padding. PC-4: ``T-count'' is a circuit property; it is replaced by the declared count of a declared circuit or by the minimal count τ. The plan's two points, the rotation family (*0 < θ < π/2*) and the local non-Clifford example, do lie in the fibre *Cl = 0* with leakages *sin θ/2* and *0*; for the stabilizer-entropy label they need not share a bin, so the pair *V_a^0, V_a^1*, with exactly equal *M_2*, is used.

The first-order witness is not necessary

Lemma. If *d = 2* and *𝒞(ρ) = ρ_00σ_0 + ρ_11σ_1* for states *σ_0, σ_1*, then *δ_full = 1/2d_tr(σ_0, σ_1)*.

Proof. The lower bound is . For the upper bound, use $\bar\sigma = (\sigma_0+\sigma_1)/2$: a referenced input gives the difference $\sum_i X_i \otimes (\sigma_i - \bar\sigma)$ with positive diagonal reference blocks X_i, $\sum_i \mathrm{Tr}\, X_i = 1$, whose trace norm is at most *d_tr(σ_0, σ_1)*. This is the argument of Theorem .

par *Non-claim.* Lemma is stated for qubit codes only. par

Theorem. Let *R = R_0R_1* and *B* be qubits, *V_0|ψ⟩ = |ψ⟩_R_0|0⟩_R_1|0⟩_B*,

$$K = \mathrm{kb}\, 11_{R_0} \otimes X_{R_1} \otimes X_B, \qquad V_\lambda = e^{-i\lambda K} V_0.$$

Then *V_0* has constant complement, and for every code pair and every residual observable the first-order coefficient *m* of Theorem vanishes; indeed *d/(dλ)𝒞_{λ}|_{λ = 0}= 0* identically. Exactly,

$$\mathscr{C}_\lambda(\rho) = \rho_{00}\,\mathrm{kb}\, 00 + \rho_{11}(\cos^2\lambda\, \mathrm{kb}\, 00 + \sin^2\lambda\, \mathrm{kb}\, 11),$$

$$\text{boxed}\ \delta_{\mathrm{full}}(V_\lambda) = \frac{\sin^2\lambda}{2}, \qquad \varepsilon^R_{\mathrm{rec}}(V_\lambda) = \frac{1-\cos\lambda}{2}.$$

So *δ_full > 0* for *0 < |λ| < π*, with leading term *λ^2/2*.

Proof. *K^2 = kb 11 ⊗ I ⊗ I*, so *e^{−iλK} = kb 00 ⊗ I + kb 11 ⊗ (cos λ I − isin λ X ⊗ X)*. Hence *V_{λ}|0⟩ = |000⟩* and *V_{λ}|1⟩ = cos λ|100⟩ − isin λ|111⟩*. Tracing *R_0R_1* gives the displayed *𝒞_{λ}*: the coherence term carries *⟨0|1⟩_R_0 = 0*. The first derivative at *0* is zero since *cos^2* and *sin^2* have zero derivative there. Lemma gives *δ_full = 1/2sin^2 λ*.

The radiation outputs are *𝒩_{λ}(ρ) = ρ_00|00⟩⟨00| + ρ_11(cos^2 λ|10⟩⟨10| + sin^2 λ|11⟩⟨11|) + cos λ(ρ_01|00⟩⟨10| + h.c.)*. The decoder *|00⟩ ↦ |0⟩, |10⟩, |11⟩ ↦ |1⟩* (completed by a fixed state) yields *Δ_{cos λ}*, whose error is *(1 − cos λ)/2* by Theorem . Conversely, the outputs of |±⟩ have trace distance *cos λ*, and the |±⟩ argument of Theorem gives *1 ≤ 2ε + cos λ*.

par *Non-claim.* The model is a finite qubit construction. It is not a continuum deformation, and the exponent *2* is a property of this coupling, not a universal law. par

Proposition. In the setting of Theorem , suppose *Tr_R [K, V_0XV_0^{†}] = 0* for every traceless *X*. For a code pair, *X = ρ_0 − ρ_1*, and *‖O‖ ≤ 1*, put

$$m_2 = \mathrm{Tr}\,[O(-\mathrm{Tr}_R\,[K,[K,V_0 X V_0^\dagger]])].$$

Then

$$\delta_{\mathrm{full}}(V_\lambda) \geq \max\{0, \frac{|m_2|\lambda^2}{8} - \frac{2}{3}\|K\|^3|\lambda|^3\}.$$

Proof. Let *f(λ) = Tr (O𝒞_{λ}(X))*. Then *f(0) = 0* by constancy, *f^{'}(0) = 0* by hypothesis, and *f^{''}(0) = m_2*. Each derivative applies one commutator with *K*, of trace-norm cost *2‖K‖*, and *‖X‖_1 ≤ 2*, so *|f^{'''}| ≤ 16‖K‖^3*. Taylor's theorem gives *|f(λ)| ≥ |m_2|λ^2/2 − 8/3‖K‖^3|λ|^3*. Apply .

par *Non-claim.* For Theorem with *O = Z_B* and *X = kb 00 − kb 11*, *m_2 = 4* and the leading coefficient *1/2* is exact. No claim is made that every first-order-blind coupling responds at second order. par

Comparison with Theorem : both families have *ε_rec^R = (1 − cos θ)/2* at the same parameter, but leakage is *sin θ/2* there and *sin^2 θ/2* here. Thus *ε_rec^R* does not determine *δ_full*, and the relation *ε_rec^R = 1/2(1 − √(1 − 4δ_full^2))* is family-specific, as stated after Theorem .

Same leakage, different resource; same resource, different leakage

Theorem. Let *d = 2*. Put

$$V_1 = V_{\mathrm{copy}}, \qquad V_2 = (T_R \otimes I_B)V_{\mathrm{copy}}, \qquad V_3 = V_T,$$

so *V_2|i⟩ = T|i⟩_R|i⟩_B* and *V_3|ψ⟩ = T|ψ⟩_R|0⟩_B*. Then *V_1* is Clifford, *V_2* and *V_3* have no Clifford dilation, and

$$\delta_{\mathrm{full}}(V_1) = \delta_{\mathrm{full}}(V_2) = \tfrac{1}{2}, \qquad \delta_{\mathrm{full}}(V_3) = 0.$$

A variant with bipartite non-local magic in a fixed syndrome is *V_2^{'}|i⟩ = |i⟩_R_0|i⟩_B_0 ⊗ |χ_{κ}⟩_R_1B_1*, *0 < κ < π/4*, with *δ_full(V_2^{'}) = 1/2*. No member of the rotation family is a valid *V_2*.

Proof. *δ_full(V_2) = δ_full(V_1)* by Proposition , and *δ_full(V_1) = 1/2* by Proposition . *V_2|+⟩ = (|00⟩ + e^{iπ/4}|11⟩)/√(2)* is not a stabilizer state, so Lemma applies; *V_3* is Proposition . For *V_2^{'}*, the complement is *Δ_Z ⊗ σ_{κ}*. The replacement *1/2I ⊗ σ_{κ}* gives *1/2*; tracing *B_1* contracts to *Δ_Z*, giving *1/2* from below. Non-local magic of *χ_{κ}* is Theorem . For the rotation family, *sin θ/2 = 1/2* forces *θ = π/2*, where *U_{π/2} = CNOT CZ* is Clifford (Theorem).

par *Non-claim.* The four-Majorana control (Theorem) is a third convention: no Pauli dictionary is declared for it, and its *δ_full = max {1/2, (1 + 2c_R)/4}* is not a magic certificate for any of *V_1, V_2, V_3*. Oai280-A's printed channels CheungOai280A, *W_good* (*δ_full = 0*, Clifford) and *W_bad* (*δ_full = 1 − d^{−2}*), are none of these isometries. par

Remark. The plan's *V_2* with *δ_full = 1/2* cannot be taken from the rotation family: *δ_full = 1/2* only at *θ = π/2*, which is Clifford. The value *1/2* is kept; *V_2* is the *T*-dressed copier, and *V_2^{'}* is a non-local-magic variant.

The channel does not determine a heat ledger

A *heat ledger* for an implementation is a declared record of the reservoir registers, their initial and final states, and the Hamiltonians against which heat is defined. Ledger registers are outside the carrier *C → R ⊗ B* and are code-independent.

Theorem. Fix an isometry *V* built from one allocated qubit *Y* in *|0⟩*, as in Section . There are two finite implementations with the same carrier isometry *V*, hence the same *𝒩*, *𝒞*, *δ_full*, and *ε_rec^R*:

• *Y* is supplied in *|0⟩*; ledger registers *Res*, *P* start in *|00⟩* and are left untouched;

• *Y* starts maximally mixed, purified by *P*, and is reset by a swap with *Res* prepared in *|0⟩* before *V* acts. The reservoir record is unchanged in (I1) and changes in (I2), from *kb 00* to *I/2*. Readout-and-drop of *B* returns *𝒩* in both and selects neither.

Proof. After the swap in (I2), *Y* is exactly *kb 00* and uncorrelated with the code, so both implementations apply the same *U* to the same input *|ψ⟩ ⊗ |0⟩_Y*. The ledger registers end in *|00⟩* and in the maximally entangled *ResP* state, respectively. Theorem gives the last sentence.

par *Non-claim.* No heat, work, or Landauer cost is computed. The reset bound of I-C-Th thm:reset, its thermal balance thm:thermal, and its access obstruction thm:access CheungICTh are source records for what a ledger would need, and En07-A prop:no-landauer-from-channel CheungEn07A is the parallel statement for channel

notation. Landauer's paper is a documentary record, not a premise Landauer1961. par

Endpoint continuity is not a Page curve

Theorem. Let $\theta, \kappa: [0, 1] \to [0, \pi/4]$ be continuous with $\theta(t) \to 0$ as $t \to 1$. For

$$V_t^{\kappa}|i\rangle = V_{\theta(t)}|i\rangle \otimes |\chi_{\kappa(t)}\rangle_{R_1B_1}$$

with $R = R_0R_1$, $B = B_0B_1$, one has $\delta_{full}(V_t^{\{\kappa\}}) = \sin \theta(t)/2 \to 0$ for every κ, and Theorem applies at $t = 1$. Yet:

• *Archive.* An append-only classical record of independent fair readouts, kept alongside, has $H(R_n) = n$ bits, strictly increasing; it never turns over.

• *Entropy history.* On the input $|0\rangle$, the radiation entropy is $h_2(\cos^2 \kappa(t))$, with h_2 the binary entropy, independently of θ; with $\kappa \equiv 0$ or $\kappa(t) = \pi/4 \sin \pi t$ the same leakage history carries different entropy histories. The leakage history therefore does not determine whether an entropy history equals the finite Haar mean of Hawk05-B thm:page or any other prescribed profile.

• *Mouth metric and endpoint.* The data are a finite family of isometries; no positive form g_k of Res01-C thm:island/thm:haar and no metric or gravitational endpoint is among them or defined from them.

Proof. The syndrome is a fixed tensor factor at each t; by the embedding argument of Theorem (b) it does not change δ_{full}. Theorem gives $\sin \theta(t)/2$. (a) The chain rule gives $H(R_{n+1}) - H(R_n) = H(X_{n+1} \mid R_n) = 1$. (b) On input $|0\rangle$, the radiation state is $kb\ 00_{R_0} \otimes Tr_{B_1} \chi_{\{\kappa(t)\}}$, of entropy $h_2(\cos^2 \kappa(t))$. (c) is a statement about the type of the data.

par *Non-claim.* This theorem only separates functions. It proves no Page curve, island, metric, or endpoint. For finite autonomous dynamics, Proposition already excludes $\delta_{full}(t) \to 0$ from a vacuum start, so the family is declared, not autonomous. I-C thm:B CheungIC, Hawk05-B thm:page CheungHawk05B, and Res01-C thm:island, thm:haar, thm:recordgap CheungRes01C are source records. Res01-C's record mean exceeds its Page target at every interior cut; that is its theorem, not this one. OpenAI Family 265 OAI265 is not this history. par

Residual Enter, then drop, returns the radiation channel

Let $\mathscr{E}_B$ be destructive computational readout of a finite B with outcome register K, as in , and put

$$\mathscr{N}_K(X) = \sum_x \mathrm{kb}\, xx_K \otimes \mathrm{Tr}_B\,[(I_R \otimes \mathrm{kb}\, xx)\mathscr{V}(X)].$$

Theorem.

• With the record retained, the access cut is $R \otimes K$: $\mathscr{N}_K: C \to R \otimes K$, and $\varepsilon_{rec}^{RK} \le \varepsilon_{rec}^{R}$, with strict inequality possible ($\varepsilon_{rec}^{RK} = 0 < 1/2 = \varepsilon_{rec}^{R}$ for the dephasing example of Section).

• Dropping the bit returns exactly $Tr_K \circ \mathscr{N}_K = \mathscr{N}$.

• The record alone does not restore the diary: $X \mapsto Tr_R \mathscr{N}_K(X)$ is classical-output, and every decoder from it has half-diamond error at least $1 - 1/d$.

• If the entire retained output is then classical, for example after a further readout of R, every decoder has half-diamond error at least $1 - 1/d$, and computational readout followed by basis preparation attains $1 - 1/d$.

Proof. (a) A decoder on R composed with Tr_K is a decoder on RK. For the example, the outcome x and the correction Z^x recover exactly; Section gives dephasing for R alone. (b) is Theorem . (c) and (d) are Theorem , which is the local form of I-C thm:A.

par *Non-claim.* The bound $1 - 1/d$ is the classical-record constant. It is distinct from $a_d = 1 - d^{-2}$ of Lemma , which is the error of a constant channel, and from the $1 - 1/d^2$ quantity of En04-B CheungEn04B. En04-B's hosted shredder cut is not used; En04-B is not registered on the site. En09-B's sharp return $1 - 1/D$ is a clock constant for unencoded CS reconstruction, CheungEn09B, and its thm:cDsharp is the robustness constant $2 - 2/D$; neither is this diary. I-C thm:A CheungIC and Hawk04-B thm:drop CheungHawk04B are source records. par

Conditional predictions (new in v3)

This section turns sixteen proposed ``predictions,'' a five-layer observational programme, eight candidate

phenomena, and a list of discovery criteria into conditional statements about the finite carrier. Each statement has the form *if a package supplies X, Y, Z, then Q*, has a complete local proof and, where one exists, an explicit witness. None is a cosmological prediction. Table maps each plan item to its result. Citations identify source records only; no proof transfer. No quantum coefficient is admitted.

Declared data and the two-point minimax lemma

Definition. A *declared package* $\mathfrak{P}$ consists of a carrier $V\colon C \to R \otimes B$ as in Section , together with any of the following, each named explicitly:

• a declared geometric factor $B \simeq G \otimes E$ and probe observables O_G (Section);

• a *prescribed host datum h* from a set $\mathscr{H}$, fixed before the code input and the same for every code input;

• *supplied functions*: a real bounded profile $\mathscr{R}$ on a host region, a carrier dictionary $\mathfrak{D}\colon \mathscr{H} \to \{\text{isometries} C \to R \otimes B\}$, a sensitivity law F, a field dictionary, or a metric map, each given as data. ``Supplied" means given; nothing in this sheet derives it. No dynamics is constructed for a host, a field, or a metric. The symbol κ denotes a coefficient symbol only; no value is given. Write $s(O) = \lambda_{\max}(O) - \lambda_{\min}(O)$ for the spread of a Hermitian O; always $s(O) \le 2\|O\|$.

Lemma. Let $\mathfrak{F}$ be a set of packages on which every admissible estimator receives the same data (a *fibre*), and let τ be a real target with $a = \inf_{\mathfrak{F}} \tau$, $b = \sup_{\mathfrak{F}} \tau$. For every deterministic or randomized report Y,

$$\sup_{\mathfrak{P} \in \mathfrak{F}} \mathbb{E}\,|Y - \tau(\mathfrak{P})| \;\ge\; \frac{b-a}{2},$$

with equality for the report $(a + b)/2$ when $b < \infty$. If $b - a = \infty$, the minimax error is infinite.

Proof. The data are identical on $\mathfrak{F}$, so Y has one law μ on the whole fibre. For $\mathfrak{P}, \mathfrak{P}^{\{\prime\}} \in \mathfrak{F}$, $\mathbb{E}_{\mu}|Y - \tau(\mathfrak{P})| + \mathbb{E}_{\mu}|Y - \tau(\mathfrak{P}^{\{\prime\}})| \ge |\tau(\mathfrak{P}) - \tau(\mathfrak{P}^{\{\prime\}})|$, so the larger term is at least half of it; take the supremum over pairs. The midpoint report has error $\max\{\tau - a, b - \tau\} \le (b - a)/2$.

par *Non-claim.* The lemma is a statement about estimators on a declared set of packages. It does not say which packages nature realizes. Big01-A thm:fibre CheungBig01A and Big04-A-coeff thm:C-adaptive CheungBig04Acoeff use the same pattern for other slots; neither is a premise here. par

Response, leakage, and the sharp constant (Predictions 14, 2)

Theorem. Let $V\colon C \to R \otimes B$ be a carrier, O a Hermitian observable on B (or on a declared factor G, applied to $\mathscr{C}_G = \mathrm{Tr}_E \mathscr{C}$), and ρ_0, ρ_1 code states.

• *Bound.*

$$|\mathrm{Tr}\,(O\,\mathscr{C}(\rho_1 - \rho_0))| \;\le s(O)\, d_{\mathrm{tr}}(\mathscr{C}(\rho_0), \mathscr{C}(\rho_1)) \;\le 2\, s(O)\, \delta_{\mathrm{full}} \;\le 4\,\|O\|\,\delta_{\mathrm{full}}.$$

Equivalently, with the normalized response $\gamma_O(V) = \sup_{\rho_0, \rho_1} |\mathrm{Tr}\,(O\mathscr{C}(\rho_1 - \rho_0))|/(2\|O\|)$ for $O \neq 0$,

$$\gamma_O \;\le \frac{s(O)}{\|O\|}\,\delta_{\mathrm{full}} \;\le 2\,\delta_{\mathrm{full}}.$$

• *Sharpness.* For the copier $V_{\pi/2}$ of and $O = Z_B$, every inequality in and is an equality: the response is 2, $s(O) = 2$, $d_{\mathrm{tr}} = 1$, $\delta_{\mathrm{full}} = 1/2$, $\gamma_O = 1$. Hence 4 is the least constant c with $|\Delta\langle O\rangle| \le c\|O\|\delta_{\mathrm{full}}$ for all carriers, and 2 is the least constant in $\gamma_O \le c\,\delta_{\mathrm{full}}$.

• *Attainment family.* If $d = 2$ and $\mathscr{C}(\rho) = \rho_{00}\sigma_0 + \rho_{11}\sigma_1$ with $\sigma_0 \neq \sigma_1$ (Lemma), the Helstrom observable $O_{*} = \Pi_{+} - \Pi_{-}$, with $\Pi_{\pm}$ the spectral projectors of $\sigma_1 - \sigma_0$ for positive and nonpositive eigenvalues, has $\|O_{*}\| = 1$ and $\gamma_{O_{*}} = 2\delta_{\mathrm{full}}$.

• *The plan's constant.* The inequality $|\Delta\langle O\rangle| \le 2\|O\|\delta_{\mathrm{full}}$ is false: the copier with $O = Z_B$ gives $2 > 1$. It becomes true, and sharp, with $\|O\|$ replaced by $s(O)$.

Proof. (a) Put $\Delta = \mathscr{C}(\rho_1) - \mathscr{C}(\rho_0) = \Delta_{+} - \Delta_{-}$ with $\Delta_{\pm} \ge 0$ of disjoint support. Since $\mathrm{Tr}\,\Delta = 0$, $\mathrm{Tr}\,\Delta_{+} = \mathrm{Tr}\,\Delta_{-} = d_{\mathrm{tr}}$. Then $\mathrm{Tr}\,O\Delta \le \lambda_{\max}\mathrm{Tr}\,\Delta_{+} - \lambda_{\min}\mathrm{Tr}\,\Delta_{-} = s(O)d_{\mathrm{tr}}$, and the same bound holds for $-\mathrm{Tr}\,O\Delta$. The second inequality is $d_{\mathrm{tr}} \le 2\,d_{\diamond}(\mathscr{C}, \mathscr{T}_{\sigma})$ for every σ, as in the proof of ; the third is $s(O) \le 2\|O\|$. Dividing by $2\|O\|$ gives . (b) $\mathscr{C}_{\pi/2}(\rho) = \rho_{00}kb\ 00 + \rho_{11}kb\ 11$; with $\rho_j = kb\ jj$, $\mathrm{Tr}\,Z\Delta = -2$, $d_{\mathrm{tr}} = 1$, and $\delta_{\mathrm{full}} = 1/2$ by Theorem . Equality in a chain of inequalities shows that no smaller constant works. (c) With

$\rho_j = kb\,jj$, $Tr\ O_{*}(\sigma_1 - \sigma_0) = \|\sigma_1 - \sigma_0\|_1 = 2d_{tr}(\sigma_0, \sigma_1) = 4\delta_{full}$ by Lemma , so $\gamma_{O_{*}} \geq 2\delta_{full}$; (a) gives the reverse inequality. (d) is (b).

par *Non-claim.* The bound concerns observables on the declared complete residual of a finite carrier. It is not a bound on any cosmological probe, and a violation in a model only shows that one of the declared hypotheses (completeness of B, the operator assigned to the probe, its normalization, finite dimension) fails there. par

Remark. PC-10: the plan's displayed $2\|O_G\|\delta_{full}$ undercounts by a factor of 2; the sharp forms are and . The plan's $\gamma \leq 2\delta_{full}$ is correct with γ normalized by $2\|O\|$, the convention of En04-B v2 CheungEn04Bv2. DC-1: En04-A v2 does not state γ; its nearest statement is the observable witness $\delta_{full} \geq w/4$ for $\|O\| \leq 1$, which is at $\|O\| = 1$.

Theorem. Suppose a package supplies a declared geometric factor $B \simeq G \otimes E$, a probe observable O_G, and code states ρ_0, ρ_1, and asserts a matter-dependent probe effect $w := Tr\,(O_G\,\mathscr{C}_G(\rho_1 - \rho_0)) \neq 0$. Put $A_{O_G} = V^{\dagger}(I_R \otimes O_G \otimes I_E)V$ on C. Then:

• $w = Tr\,(A_{O_G}(\rho_1 - \rho_0))$; hence an effect exists for some pair if and only if A_{O_G} is not scalar;

• $\delta_{full} \geq |w|/(2s(O_G)) \geq |w|/(4\|O_G\|)$;

• $\mathscr{C}$ is not constant, so whole-code recovery from R is not exact (Theorems and). Witness: for V_{θ} of with $G = B$ and $O_G = Z$, $w = -2\sin^2\theta$, nonzero for $0 < \theta \leq \pi/2$.

Proof. (a) $Tr\,(O_G\mathscr{C}_G(\rho)) = Tr\,((I \otimes O_G \otimes I)V\rho V^{\dagger}) = Tr\,(A_{O_G}\rho)$. If $A_{O_G} = cI$, $w = cTr\,(\rho_1 - \rho_0) = 0$; if not, two eigenvectors with distinct eigenvalues give $w \neq 0$. (b) is . (c) $\delta_{full} > 0$ by (b), so $\mathscr{C}$ is not constant (Lemma). The witness: $\langle Z\rangle$ is 1 on $|0\rangle$ and $\cos 2\theta$ on $|b_{\theta}\rangle$.

par *Non-claim.* The theorem states what a probe effect requires; it does not say that any physical observable is represented by O_G, and it does not identify G with geometry (Theorem). par

Theorem. Let $B = E \otimes G$ be two qubits and $V_{corr}|j\rangle = |j\rangle_R|\Phi_j\rangle_{EG}$ with $|\Phi_0\rangle = |\Phi^{+}\rangle$, $|\Phi_1\rangle = |\Phi^{-}\rangle$. Then:

• $\mathscr{C}_E = \mathscr{C}_G = \mathscr{T}_{I/2}$: both marginals are constant, so every local witness $Tr\,((O_E \otimes I)\mathscr{C}(\cdot))$, $Tr\,((I \otimes O_G)\mathscr{C}(\cdot))$ vanishes;

• $\mathscr{C}_{EG}(\rho) = \rho_{00}|\Phi^{+}\rangle\langle\Phi^{+}| + \rho_{11}|\Phi^{-}\rangle\langle\Phi^{-}|$ is not constant, $\delta_{full} = 1/2$, and $\varepsilon_{rec}^R = 1/2$;

• the product observable $X_E \otimes X_G$ has response 2 on $\rho_j = kb\,jj$, and

$$\sup_{\|O_E\|,\|O_G\| \leq 1} \sup_{\rho_0,\rho_1} |\mathrm{Tr}\,((O_E \otimes O_G)\mathscr{C}(\rho_1 - \rho_0))| = 2,$$

which is the sharp value $4\|O\|\delta_{full}$ of Theorem .

Proof. (a) Both Bell vectors have maximally mixed marginals, and the coherences ρ_{01} carry $\langle 0|1\rangle_R = 0$. (b) The two outputs are orthogonal pure states, so Lemma gives $\delta_{full} = 1/2$. The radiation channel is the complete dephasing Δ_0, whose error is $1/2$ by Theorem with $\cos\theta = 0$. (c) $\langle X \otimes X\rangle_{\Phi^{\pm}} = \pm 1$. For $\|O_E \otimes O_G\| \leq 1$, gives at most $4 \cdot 1/2 = 2$.

par *Non-claim.* E and G are declared labels. The same carrier with E renamed ``auxiliary memory'' has identical channels, so a correlation witness alone does not show that a correlation is governed by a geometric law, and no gravitational correlation is asserted. par

Sampling a prescribed profile (Predictions 4, 5)

Definition. A *declared finite compression* consists of a prescribed host region U with a measure, a supplied bounded real profile $\mathscr{R}$ on U (the same for every code input), orthonormal $f_0, f_1 \in L^2(U)$, and the identification $G = span\,\{|g_0\rangle, |g_1\rangle\}$ with

$$O_{\mathscr{R}} = \sum_{a,b \in \{0,1\}} \langle f_a, \mathscr{R} f_b\rangle\, |g_a\rangle\langle g_b|, \qquad \kappa_a = \langle f_a, \mathscr{R} f_a\rangle, \quad c = \langle f_0, \mathscr{R} f_1\rangle.$$

The probe carrier is V_{η} of with $\theta = \eta$, $B = G$, and $|b_{\eta}\rangle = \cos\eta|g_0\rangle + \sin\eta|g_1\rangle$.

Theorem. If $\mathscr{R} = \mathscr{R}_0$ is constant almost everywhere on $supp\ f_0 \cup supp\ f_1$, then $O_{\mathscr{R}} = \mathscr{R}_0 I_G$ and every carrier $V: C \to R \otimes (G \otimes E)$ has zero $O_{\mathscr{R}}$-witness for every pair of code states. More generally, $O_{\mathscr{R}}$ is scalar if and only if $\kappa_0 = \kappa_1$ and $c = 0$. The blindness is specific to $O_{\mathscr{R}}$: for $V_{\pi/2}$, $\delta_{full} = 1/2$ and the

probe $|g_1\rangle\langle g_1|$ has response *1*.

Proof. $\langle f_a, \mathscr{R}_0 f_b\rangle = \mathscr{R}_0 \delta_{ab}$. A scalar compression has zero witness by Theorem (a). A Hermitian 2×2 matrix is scalar if and only if its diagonal entries agree and its off-diagonal entry vanishes. For $V_{\pi/2}$, $\mathscr{C}(kb\ 11) = |g_1\rangle\langle g_1|$ and $\mathscr{C}(kb\ 00) = |g_0\rangle\langle g_0|$.

par *Non-claim.* ``Constant profile'' means a constant supplied function on a prescribed host. No statement is made about de Sitter space, about a physical scalar-curvature probe, or about which other observables a physical theory would provide. par

Theorem. For the declared compression and V_{η}, with $\rho_j = kb\ jj$,

$$w = \operatorname{Tr}\left(O_{\mathscr{R}}\,\mathscr{C}(\rho_1 - \rho_0)\right) = (\kappa_1 - \kappa_0)\sin^2\eta + \sin 2\eta\,\operatorname{Re} c.$$

• If f_0, f_1 have disjoint supports, then $c = 0$ and $w = (\kappa_1 - \kappa_0)\sin^2\eta$.

• $\delta_{full}(V_{\eta}) = \tfrac{1}{2}\sin\eta$, and $|w| \le s(O_{\mathscr{R}})\sin\eta = 2s(O_{\mathscr{R}})\delta_{full}$, as Theorem requires. For $c = 0$, $\gamma_{O_{\mathscr{R}}} = |\kappa_1 - \kappa_0|\sin^2\eta/(2\max\{|\kappa_0|, |\kappa_1|\})$.

• *Sampling, not backreaction.* The profile is the same function for both inputs, so the host-level difference $\mathscr{R}^{(1)} - \mathscr{R}^{(0)}$ is identically zero, while $w \neq 0$ whenever $\kappa_1 \neq \kappa_0$, $c = 0$, and $\sin\eta \neq 0$. A backreaction statement needs an input-dependent map $\rho \mapsto \mathscr{R}_{\rho}$; none is in the data.

Proof. $\mathscr{C}(\rho_0) = |g_0\rangle\langle g_0|$ and $\mathscr{C}(\rho_1) = |b_{\eta}\rangle\langle b_{\eta}|$, so $w = \cos^2\eta\,\kappa_0 + \sin^2\eta\,\kappa_1 + 2\sin\eta\cos\eta \operatorname{Re} c - \kappa_0$, which is . (a) The integrand of c vanishes. (b) Theorem gives δ_{full}, and the bound. For $c = 0$, $O_{\mathscr{R}} = \operatorname{diag}(\kappa_0, \kappa_1)$ and the code response is $\operatorname{Tr}(A\rho)$ with $A = \operatorname{diag}(\kappa_0, \kappa_0\cos^2\eta + \kappa_1\sin^2\eta)$, whose spread is $|\kappa_1 - \kappa_0|\sin^2\eta$. (c) is a property of the declared data.

par *Non-claim.* κ_a are compressions of a supplied function; they are not a curvature, a coupling, or the coefficient κ. The theorem does not say that inhomogeneous cosmological backgrounds can be probed, and it constructs no host dynamics. par

Remark. PC-11: the plan's witness ``$\propto (\kappa_1 - \kappa_0)\sin^2\eta$'' omits the cross term $\sin 2\eta \operatorname{Re} c$; it holds as stated when $c = 0$, for example for disjoint supports. En04-B v2 thm:leak-exact CheungEn04Bv2 writes the rod response as $\gamma_O = |\lambda_1 - \lambda_0|\sin(\theta_{-} + \theta_{+})\sin\Delta/(2\max\{|\lambda_0|, |\lambda_1|\})$; at $\theta_{-} = 0$, $\theta_{+} = \eta$, $\lambda_a = \kappa_a$ this is the normalized form in Theorem (b). The unnormalized En04-A witness is . No step of the proof uses En04-B.

Fibres and minimax errors (Predictions 1, 6, 15, 7, 16, 3)
Theorem.

• *Fixed declared geometry.* Let $V_{\theta}^{\otimes G} = V_{\theta} \otimes |g_0\rangle_G$ with residual $B \otimes G$. For every $\theta \in [0, \pi/2]$, $\mathscr{C}_G = \mathscr{T}_{|g_0\rangle\langle g_0|}$ and the host datum are the same, while $\delta_{full}(V_{\theta}^{\otimes G}) = \tfrac{1}{2}\sin\theta$ takes every value in $[0, 1/2]$. The minimax error for δ_{full} from these data is $1/4$.

• *Fixed host datum, all carriers.* Fix $h \in \mathscr{H}$ and spaces with $\dim R, \dim B \ge d$. The set $\{\delta_{full}(V)\}$ over isometries $V: C \to R \otimes B$ is exactly $[0, a_d]$, $a_d = 1 - d^{-2}$; the minimax error for δ_{full} from $(h, \dim R, \dim B)$ is $a_d/2$. The same fibre contains both $\varepsilon_{rec}^{R} = 0$ and $\varepsilon_{rec}^{R} = a_d$.

• *Fixed carrier, two hosts.* For hosts $h_1 \neq h_2$ and any real host coordinate φ, the packages $(h_1, V), (h_2, V)$ have identical channel data; any estimator of φ from channel data has minimax error at least $|\varphi(h_1) - \varphi(h_2)|/2$.

• *Conditional positive form.* If a package supplies a carrier dictionary $\mathfrak{D}$, then $h \mapsto \delta_{full}(\mathfrak{D}(h))$ is a function, and the error in (b) is 0 on the image of $\mathfrak{D}$.

Proof. (a) Tr_{RB} of $V_{\theta}^{\otimes G}\rho V^{\{G\dagger\}}$ is $|g_0\rangle\langle g_0|$; adjoining a fixed pure factor does not change the leakage (contraction in both directions, as in the proof of Theorem); then Theorem and Lemma . (b) $V_{good}|i\rangle = |i\rangle_R|0\rangle_B$ has constant complement ($\delta_{full} = 0$, $\varepsilon_{rec}^{R} = 0$); $V_{bad}|i\rangle = |0\rangle_R|i\rangle_B$ has $\mathscr{C} = id$ and $\mathscr{N}$ constant, so $\delta_{full} = a_d$ and $\varepsilon_{rec}^{R} = a_d$ by Lemma , which also gives the upper bound a_d. Every isometry is UV_{good} for a unitary U on $R \otimes B$, and the unitary group is path connected, so isometries form a connected set. By Lemma , δ_{full} is continuous in V; the intermediate value theorem gives every value in $[0, a_d]$. Lemma gives $a_d/2$. (c) The host datum is not an argument of $\mathscr{C}$ or $\mathscr{N}$; apply Lemma . (d) is immediate.

par *Non-claim.* Hosts are labels in a declared set. The theorem does not say that two cosmologies share a leakage

history, that a Hubble rate, curvature, or horizon area is unconstrained in nature, or that any carrier dictionary exists. par

Remark. PC-15: the range *[0, a_d]* in (b) needs *dim R, dim B ≥ d*; for smaller spaces the range is an interval *[0, m]* with *m ≤ a_d*, and the minimax error is *m/2*. Big01-A thm:fibre CheungBig01A is the same pattern for a different host slot.

Theorem. On the family *V_t^{κ}* of Theorem , fix the schedule *θ(·)*. Then at every *t* the leakage is *1/2sin θ(t)* for every schedule *κ(·)*, while the input-*|0⟩* radiation entropy *h_2(cos^2 κ(t))* takes every value in *[0, 1]* bit as *κ(t)* ranges over *[0, π/4]*. Hence any map from leakage histories to entropy histories has minimax error at least *1/2* bit at every *t*. Likewise, a supplied host function *H(·)* adjoined to the package is unconstrained by the leakage history: for *H_1 ≠ H_2* the error is at least *sup_t |H_1(t) − H_2(t)|/2*. If a package supplies *κ(·)*, the entropy history is fixed.

Proof. Theorem gives the leakage and the entropy formula; *κ ↦ h_2(cos^2 κ)* is continuous and increases from *0* to *1* on *[0, π/4]*. Apply Lemma pointwise in *t*, and to *H(t)* as a host datum as in Theorem (c).

par *Non-claim. H* is a supplied function, not an expansion rate. No Page curve, cosmological entropy, or expansion history is derived. par

Theorem. Let *K ⊆ ℝ* be a range of a coefficient symbol *κ* and suppose a package's admitted data have a law *P_{κ}*.

• If *P_{κ}* does not depend on *κ*, every estimator has minimax error at least *(sup K − inf K)/2*, infinite for *K = (0, ∞)*.

• If a package supplies a probe *O_G*, a code state *ρ_{*}*, and a sensitivity law *Tr (O_G𝒞_G(ρ_{*})) = F(κ)* with *F* injective on *K*, then exact data determine *κ = F^{−1}(Tr (O_G𝒞_G(ρ_{*})))*. For example, *F(κ) = κw_0* with *w_0 ≠ 0* gives *κ = y/w_0* symbolically.

• The quantities *δ_full*, *ε_rec^R*, and every witness of this sheet are defined without *κ*; the finite carriers here supply no *F*.

Proof. (a) is Lemma . (b) is the definition of injectivity. (c) *κ* does not occur in , , or in any declared carrier.

par *Non-claim.* No value of *κ* is given and no sensitivity law is asserted. Big04-A-coeff thm:C-adaptive CheungBig04Acoeff is the parallel statement for its coefficient slot and is not a premise. par

Theorem. Use the labels of Theorem (Clifford flag *Cl*, minimal *T*-count *τ*, stabilizer entropy *M_2* of the Choi vector) and the response *Δ⟨Z_B⟩ = Tr (Z_B𝒞(kb 11 − kb 00))*.

• *Equal label, zero and nonzero response:* Theorem .

• *Equal label, opposite response. V_cp = V_{π/2}* and *(I_R ⊗ X_B)V_cp* both have *Cl = 1*, *τ = 0*, *M_2 = 0*, and *Δ⟨Z_B⟩ = −2* and *+2*.

• *Equal response, different label. V_cp* and *(T_R ⊗ I_B)V_cp* have the same complementary channel, hence the same response for every observable and every pair, while *M_2 = 0* versus *log_2 (4/3)*, *Cl = 1* versus *0*, and *τ = 0* versus *1*.

• Consequently there is no function *F* with response *= F(label)*, and no function with label *= F(response)*; the minimax error for *M_2* from the complementary channel is at least *1/2log_2 (4/3)*.

Proof. (b) *X_B* is a Pauli on the output; Clifford dilations compose, the Choi vector changes by a local Pauli, and *M_2* is invariant under local Paulis. *𝒞(kb jj) = kb jj* for *V_cp* and *|1 − j⟩⟨1 − j|* after *X_B*. (c) A unitary on *R* does not change *Tr_R*. The Choi vector of *(T_R ⊗ I)V_cp* is *(|000⟩ + e^{iπ/4}|111⟩)/√(2)*; its nonzero Pauli expectations are *±1* on the *4 Z*-strings and *±2^{−1/2}* on *8* strings with *X* or *Y* on all three qubits, so *∑_P ⟨P⟩^4 = 4 + 8 · 1/4 = 6* and *M_2 = −log_2 (6/8) = log_2 (4/3)*. It is not a stabilizer state, so *Cl = 0* by Lemma , *τ ≥ 1*, and the one *T* gate gives *τ = 1*. (d) follows, with Lemma for the last claim.

par *Non-claim.* The labels are declared finite resource labels; no statement relates magic to curvature, and no functional *𝒲* or coupling *κ* is proposed. par

Records, echoes, clocks, and order (Predictions 8–11)
Theorem.

• If the retained output is entirely classical (a measure-and-prepare channel), every decoder has error at least $1 - 1/d$ (Theorem), and computational readout attains it (Theorem).

• $1 - 1/d < a_d = 1 - d^{-2}$ for $d \geq 2$; a_d is the error of an input-independent output (Lemma), a different task.

• A classical record that in addition stores a declared host label h exactly still has recovery error $1 - 1/d$ when the code readout is computational: parameter information does not lower the floor.

Proof. (a),(b) are the cited results and arithmetic. (c) The label register is in an input-independent state; appending it is a channel and discarding it is a channel, so the recovery error is unchanged in both directions.

par *Non-claim.* The records are finite registers. No statement is made about CMB data, detector logs, or which parameters an actual observation determines. par

Theorem. Let $\alpha \in A \subseteq \mathbb{R}$ be a declared host parameter and $\mathscr{K}_{\alpha}(\rho) = Tr\,(\rho)\,\omega_{\alpha}$.

• For every decoder $\mathscr{D}$, $F_e(\mathscr{D}\mathscr{K}_{\alpha}) = 1/d^2$ and $d_{\diamond}(\mathscr{D}\mathscr{K}_{\alpha}, id) \geq a_d$; the recovery error is exactly a_d.

• One use distinguishes α_1, α_2 with optimal success probability $1/2(1 + d_{tr}(\omega_{\alpha_1}, \omega_{\alpha_2}))$.

• If $\|S\| \leq 1$ and $s(\alpha) = Tr\,(S\omega_{\alpha})$ satisfies $|s(\alpha) - s(\alpha')| \geq c|\alpha - \alpha'|$ on A with $c > 0$, then $d_{tr}(\omega_{\alpha}, \omega_{\alpha'}) \geq c|\alpha - \alpha'|/2$, s is injective on A, and for $\hat{s} \in s(A)$ the estimator $\hat{\alpha} = s^{-1}(\hat{s})$ satisfies $|\hat{\alpha} - \alpha| \leq |\hat{s} - s(\alpha)|/c$.

• Witness: $\omega_{\alpha} = |\psi_{\alpha}\rangle\langle\psi_{\alpha}|$, $|\psi_{\alpha}\rangle = \cos\alpha|0\rangle + \sin\alpha|1\rangle$, $S = X$, $A = [0, \pi/8]$: $s = \sin 2\alpha$, $c = \sqrt{2}$, $d_{tr}(\omega_{\alpha}, \omega_{\alpha'}) = |\sin(\alpha - \alpha')|$.

Proof. (a) $\mathscr{D}\mathscr{K}_{\alpha} = \mathscr{T}_{\mathscr{D}(\omega_{\alpha})}$, whose Choi state is $I/d \otimes \mathscr{D}(\omega_{\alpha})$; its overlap with $|\Phi_d\rangle$ is d^{-2}. Lemma gives the distance bound and the universal upper bound. (b) Helstrom. (c) $|s(\alpha) - s(\alpha')| \leq 2\|S\|d_{tr}$. Co-Lipschitz maps are injective, and $|\hat{\alpha} - \alpha| \leq |s(\hat{\alpha}) - s(\alpha)|/c$. (d) $s'(\alpha) = 2\cos 2\alpha \geq \sqrt{2}$ on A; mean value theorem; pure-state trace distance $\sqrt{(1 - |\langle\psi_{\alpha}|\psi_{\alpha'}\rangle|^2)}$.

par *Non-claim.* α is a declared parameter and ω_{α} a declared state; no cosmological echo or geometry sensitivity is asserted. par

Remark. PC-12: ``$|\Delta\alpha| \lesssim |\Delta s|/c$'' holds exactly as (c), provided both values lie in the interval A where s is co-Lipschitz; outside A it can fail (for $S = Z$, $s = \cos 2\alpha$ is flat at 0). DC-3: the echo reading is En04-B and I-C material, not an En04-A v2 theorem; it is proved here locally.

Theorem. Let $V_K|j\rangle = |j\rangle_R|\tau_j\rangle_K$ with the clock K in the residual.

• $\mathscr{C}(\rho) = \sum_j \rho_{jj}|\tau_j\rangle\langle\tau_j|$. For $d = 2$, $\delta_{full} = 1/2\sqrt{(1 - |\langle\tau_0|\tau_1\rangle|^2)}$: $1/2$ for orthogonal clock states and 0 for a neutral clock $\tau_0 = \tau_1$.

• For d orthogonal clock states, $\delta_{full} = 1 - 1/d$.

• For $d = 2$, $\varepsilon_{rec}^R = 1/2(1 - |\langle\tau_0|\tau_1\rangle|)$; if K is retained quantum mechanically with R, recovery is exact.

• An orthogonal qubit clock has exactly the complementary channel of the copier $V_{\pi/2}$; no residual observable distinguishes the clock from an environment copy.

Proof. (a) The coherences carry $\langle j|k\rangle_R = 0$. Lemma with pure σ_j gives the formula. (b) Up to the isometry $|j\rangle \mapsto |\tau_j\rangle$ and its left inverse (both contractions), $\mathscr{C}$ is the complete dephasing Δ_d. Lower bound: on $|\Phi_d\rangle$, Δ_d gives $1/d\sum_i |ii\rangle\langle ii|$, which has weight 1 on $P = \sum_i |ii\rangle\langle ii|$, while $I/d \otimes \sigma$ has weight $1/d\sum_i \sigma_{ii} = 1/d$. Upper bound: with Weyl operators $W_{ab} = X^aZ^b$, $\mathscr{T}_{I/d} = d^{-2}\sum_{a,b} Ad_{W_{ab}}$ and $\Delta_d = d^{-1}\sum_b Ad_{Z^b}$, so $\Delta_d - \mathscr{T}_{I/d} = (d^{-1} - d^{-2})\sum_b Ad_{Z^b} - d^{-2}\sum_{a \neq 0, b} Ad_{W_{ab}}$, whose half-diamond norm is at most $1/2(d(d^{-1} - d^{-2}) + (d^2 - d)d^{-2}) = 1 - 1/d$. (c) $\mathscr{N}(\rho) = \sum_{jk} \rho_{jk}\langle\tau_k|\tau_j\rangle kb\,jk$, which is $\Delta_{|\langle\tau_0|\tau_1\rangle|}$ after a diagonal unitary on R; Theorem gives the error. With K retained, V_K itself is an isometry into the accessible system. (d) Both complements are $\rho \mapsto \rho_{00}kb\,00 + \rho_{11}kb\,11$.

par *Non-claim.* K is a declared register. No relational-time model, physical clock, or gravitational information loss is described. par

Remark. PC-13: leakage $1/2$ needs orthogonal qubit clock states; the general values are (a) and (b). The plan's

five options are placed as follows: an independent clock is $\tau_0 = \tau_1$ ($\delta_{full} = 0$); an entangled clock is (a)–(b); a destructively measured clock is Theorem ; a clock retained quantum mechanically is the exact case of (c); a clock in the residual partition is the setting of (a).

Theorem. For the carrier $V_{\lambda} = e^{-i\lambda K}V_0$ of Theorem , the first derivative of $\mathscr{C}_{\lambda}$ at $\lambda = 0$ vanishes, yet

$$\delta_{\text{full}} = \tfrac{1}{2}\sin^2\lambda, \qquad \varepsilon^{R}_{\text{rec}} = \tfrac{1}{2}(1-\cos\lambda), \qquad \operatorname{Tr}\left(Z_B \mathscr{C}_{\lambda}(\text{kb}\,00 - \text{kb}\,11)\right) = 2\sin^2\lambda,$$

all even in λ, with leading terms $\lambda^2/2$, $\lambda^2/4$, $2\lambda^2$. Hence a vanishing first-order witness does not imply a constant complement, and the order of the leading response is a property of the supplied coupling.

Proof. Theorem ; $\langle Z_B \rangle$ is 1 on input $|0\rangle$ and $\cos^2\lambda - \sin^2\lambda$ on input $|1\rangle$.

par *Non-claim.* The exponent concerns channel quantities of a finite carrier. No statement $\Delta g \sim \lambda^2$, $\Delta R \sim \lambda^2$, or $\Delta H \sim \lambda^2$ is made (PC-17). par

Layers, the constraint chain, and requirements (S 17, S 19)

Definition. For a package and code states ρ_0, ρ_1:

L0 (resource) the carrier has a nonzero declared resource label ($Cl = 0$ or $M_2 > 0$);

L1 (residual response) $\mathscr{C}(\rho_1) \neq \mathscr{C}(\rho_0)$;

L2 (probe response) a declared O_G has $w(O_G; \rho_0, \rho_1) \neq 0$;

L3 (field response) a supplied field dictionary assigns states ω_j of a supplied field algebra to ρ_j, and $\omega_1(\Phi(f)) \neq \omega_0(\Phi(f))$ for a smeared field $\Phi(f)$;

L4 (metric response) a supplied metric map assigns metrics $g^{(j)}$ on a host to ρ_j, and $g^{(1)} \neq g^{(0)}$;

L5 (same-model law) a supplied law relates the L4 difference to independently defined L3 data, e.g. $G[g^{(1)}] - G[g^{(0)}] = \kappa\, \Delta T[\omega_0, \omega_1]$ with ΔT defined from the field algebra alone.

Proposition. For each $k \in \{0, 1, 2, 3, 4\}$ there is a package satisfying Lk and not L$(k+1)$: -1

• $V_T = (T_R \otimes I)V_{good}$: $M_2 = \log_2(4/3)$, $\delta_{full} = 0$.

• $V_{cp} \otimes |g_0\rangle_G$ with declared G: $\mathscr{C}$ nonconstant ($\delta_{full} = 1/2$), every O_G has zero witness.

• Supplied one-particle space $\mathfrak{h} \ni g_0, g_1$ (orthonormal), Fock dictionary $\rho_0 \mapsto a^\dagger(g_0)\Omega$, $\rho_1 \mapsto a^\dagger(\cos\eta\, g_0 + \sin\eta\, g_1)\Omega$, and declared probe $N(g_1) = a^\dagger(g_1)a(g_1)$ (the probe $|g_1\rangle\langle g_1|$ of V_{η}): the probe responds with $\sin^2\eta$, while $\omega_j(\Phi(f)) = 0$ for every f and both j.

• Coherent dictionary $\rho_0 \mapsto \Omega$, $\rho_1 \mapsto W(\alpha)\Omega$ on a prescribed host with metric g_{host}: $\omega_1(\Phi(f)) = 2\mathrm{Re}\,\langle f, \alpha\rangle \neq \omega_0(\Phi(f)) = 0$ for some f, while $g^{(1)} = g^{(0)} = g_{host}$.

• Any supplied metric pair $g^{(0)} \neq g^{(1)}$ and supplied operator G: for every symbol $\kappa \neq 0$, $\Delta T_{\kappa} := (G[g^{(1)}] - G[g^{(0)}])/\kappa$ satisfies the displayed relation, so the metric data select neither a law nor κ; L5 requires ΔT to be supplied independently, and it is not. In the reverse direction, L2 implies L1 (Theorem), while L1 does not imply L0 (the copier is Clifford with $M_2 = 0$).

Proof. (0) T_R does not change Tr_R, so $\mathscr{C}$ is constant; the Choi vector is $(|00\rangle + e^{i\pi/4}|11\rangle)/\sqrt{2}$ up to a fixed factor, with $M_2 = \log_2(4/3)$ by the count in Theorem . (1) $\mathscr{C}_G \equiv |g_0\rangle\langle g_0|$; then Theorem (a). (2) $\Phi(f) = a(f) + a^\dagger(f)$ changes particle number by ± 1, so its expectation in a one-particle state is 0; and $\langle N(g_1)\rangle$ is $|\langle g_1, \cdot\rangle|^2$ of the one-particle vector. (3) $a(f)W(\alpha)\Omega = \langle f, \alpha\rangle W(\alpha)\Omega$; take $f = \alpha$. A prescribed host has an input-independent metric by Definition (ii). (4) Algebra. The reverse statements are Theorem and Theorem (b).

par *Non-claim.* The field algebra, the dictionaries, and the metrics are supplied data on prescribed hosts; no dynamics, Einstein equation, or backreaction is constructed. Layer L3 is defined by one-point data; second moments of $\Phi(f)$ do differ in example (2) (PC-14). par

Definition. A cosmological reading of this sheet would need: R1 a physical spacetime; R2 a defined matter Hilbert space; R3 a complete preparation map; R4 a field-to-carrier map; R5 a declared access partition; R6 a geometric observable algebra; R7 a dynamical matter–geometry coupling; R8 a derived metric response; R9 a controlled approximation or continuum limit; R10 a prediction distinguishable from standard cosmology. This sheet supplies R2, R3, R5, and R6 only as declared finite data ($C, V, R: B, O_G$); it supplies none of R1, R4, R7,

R8, R9, R10.

Theorem. In the chain

$$\text{resource} \;\nRightarrow\; \text{leakage} \;\nRightarrow\; \text{?probe response}$$
$$\nRightarrow\; \text{field response?} \;\nRightarrow\; \text{metric backreaction (L4 and L5)},$$

each arrow fails on the corresponding example of Proposition , and holds once the following law is added:

• resource → leakage: a coupling witness, i.e. a nonscalar compression $V^{\dagger}(I \otimes O)V$ for some residual O (Theorem (a)); the resource label does not enter;

• leakage → probe: a declared factor G and O_G with nonscalar A_O_G;

• probe → field: a supplied field dictionary under which $\omega_j(\Phi(f)) = Tr\,(O_G \mathscr{C}_G(\rho_j))$ for some f;

• field → metric: a supplied input-dependent metric map $g[\rho] = \mathfrak{G}(\omega_{\rho}(\Phi(\cdot)))$ with $\mathfrak{G}$ injective; and for L5 a supplied same-model law with ΔT defined independently and a fixed κ. None of these laws is supplied in this sheet (Definition).

Proof. Failure: Proposition . Sufficiency: (i) a nonscalar compression gives a nonzero witness and so $\delta_{full} > 0$ by Theorem ; (ii) the same; (iii) the field difference equals the probe difference; (iv) injectivity of $\mathfrak{G}$ turns different one-point data into different metrics, and L5 is then a checkable identity rather than a definition of ΔT.

par *Non-claim.* The added laws are listed, not proposed. The chain is a structure of constraints on declared packages, not a cosmological prediction; any observational content is not derived. par

Candidate phenomena (S 18): conjectures and finite analogues

Conjecture. The following are labelled conjectures in the plan's sense; this sheet does not prove or support them as cosmology. (A) correlation-only matter–geometry response; (B) quadratic backreaction ΔR, $\Delta H \propto \lambda^2$; (C) leakage without metric change; (D) recovery transitions at fixed geometry; (E) constant-curvature blindness of scalar probes; (F) clock-induced apparent information loss; (G) distinct Page-like curves with one leakage curve; (H) nonlocal first signatures.

Proposition. Each item has a finite analogue on declared packages:

A, H V_{corr}: local witnesses zero, the joint $X_E \otimes X_G$ witness 2 (Theorem);

B $\delta_{full} = 1/2 \sin^2 \lambda$ with vanishing first derivative (Theorem);

C V_{cp} on a prescribed host: $\delta_{full} = 1/2$, host metric unchanged by definition;

D $V|j\rangle = |j\rangle_R|j\rangle_{B_1}|g_0\rangle_G$: with G and the host fixed, $\varepsilon_{rec}^R = 1/2$ and $\varepsilon_{rec}^{RB_1} = 0$ when B_1 joins the access;

E Theorem ;

F Theorem (d);

G Theorem and Theorem .

Proof. D: with access R, $\mathcal{N}$ is the complete dephasing, error $1/2$ (Theorem , $\cos\theta = 0$); with access RB_1, the remaining complement is $|g_0\rangle\langle g_0|$, constant, so recovery is exact (Theorem). C: Theorem and Definition (ii). The other items are the cited results.

par *Non-claim.* Each analogue is a finite channel statement. The conjectures keep their status; their cosmological reading is not claimed, supported, or tested here. par

Completed program and destination ledger

toprule Declared program or condition Proved channel statement What is not inferred

midrule endfirsthead toprule Declared program or condition Proved channel statement What is not inferred

midrule endhead bottomrule endfoot

Identity or tested Clifford baseline Complete complement constant; quantum radiation return is exact. No geometric model follows from Cliffordness.

Clifford CNOT copying body $\delta_{full} = \varepsilon_{rec}^R = 1/2$; logical basis information enters B. Magic is not necessary

for residual response.

Local non-Clifford logical body Complement constant; inverse logical unitary recovers exactly. A non-Clifford gate does not force leakage.

Fixed non-local magical syndrome Complement fixed mixed; logical register remains exactly recoverable. Bipartite non-local magic is not sufficient for response.

Controlled-R_y body $\delta_{full} = \sin\theta/2$; $\varepsilon_{rec}^{R} = (1 - \cos\theta)/2$. The formulas are model-specific, not universal magic laws.

Four-Majorana control $\varepsilon_{rec}^{R} = (3 - z_R)/4 \geq 1/2$; complete output reversible. No scrambling, magic, or gravitational certificate.

Local quantum decoder Returns the code register; work is explicitly consumed. Enter on the diary is not the decoder.

Residual Enter with record supplied Access becomes *RK*; outcome-assisted correction may help. Bits alone do not replace an unknown diary.

Residual Enter then dropBit Observable channel is exactly $\mathcal{N}$. No permanent physical deletion or additional radiation erasure.

Constant radiation Complete residual alone exactly recovers the code. Global preservation alone does not imply this localization.

Nonconstant complete residual Some logical information is distinguished by *B*. Residual-only recovery of the whole code need not hold.

Constant endpoint complete residual Exact endpoint radiation recovery, with access-preserving outputs. No gravitational endpoint without a physical construction.

New in version 2

Declared factor $B \simeq G \otimes E$ Blind *G* iff exact recovery from *RE*; adjoined factor always blind (Thm.). Response never forces a geometric factor.

Bridge hypotheses (i)–(iii) (ii) gives (C1); (i)+(ii) give (C2); all three give (C3) (Thm.). The certificate is idle for the printed conclusion.

One magic label Each fibre contains $\delta_{full} = 0$ and $\delta_{full} = 1/2$; indicator error $1/2$ (Thm.). No label selects leakage.

First-order-blind coupling $\delta_{full} = 1/2\sin^2\lambda$, $\varepsilon_{rec}^{R} = 1/2(1 - \cos\lambda)$ (Thm.). A zero commutator witness does not certify nonresponse.

Resource pairs $\delta_{full}(V_1) = \delta_{full}(V_2) = 1/2$, $\delta_{full}(V_3) = 0$ (Thm.). Leakage is not a resource measure.

Two implementations, one channel Different reservoir records (Thm.). No heat is computed.

Leakage history $\to 0$ Entropy and record histories not fixed (Thm.). No Page curve, mouth metric, or endpoint.

Residual Enter, record kept or dropped Cut *RK*; drop gives $\mathcal{N}$; classical-only error $\geq 1 - 1/d$ (Thm.). Not the shredder cut; not a clock constant.

toprule Plan step Completed or corrected deliverable

midrule 1 Finite prepared-carrier interface and reference-preserving destructive Born instrument. No field preparation is derived from typing.

2 Global restore and exact constant-complement criterion. A Clifford baseline requires the Pauli test, not its label.

3 The unconditional magic implication is disproved. A non-Clifford responding family and a general response witness are constructed instead.

4 Joint preservation, residual-only localization, observation, and operational forgetting are separated. No deletion theorem is claimed.

5 Access-preserving endpoint continuity and arbitrarily late finite-proxy returns are proved without promoting termination to evaporation.

6 OpenAI and XIV/XV/XVI comparisons remain documentary separations; they supply no recovery, magic, geometry, or admission premise.

v2 Eight criteria and typed refusals (Section); verdicts in Appendix .

v3 Conditional predictions, layers, constraint chain, and conjectures with finite analogues (Section); verdicts in Appendix .

bottomrule

Conclusion

The generic implication from non-Clifford or bipartite non-local magic to a nonconstant complete residual is false. The converse implication is false as well. The exact invariant governing whole-code radiation recovery is constancy of the complete complementary channel.

A specified non-Clifford deformation can produce residual response: the controlled-rotation construction proves this with exact distances, and the first-order observable theorem supplies a general sufficient condition. The coupling witness, not the resource label alone, establishes the conclusion.

Residual response is not geometric backreaction. Version 2 makes this a criterion (Theorems and). A geometric interpretation needs a justified subsystem or observable dictionary, a matter-input identification, and same-model physical dynamics. The supplied plan cannot replace these hypotheses by terminology.

Enter remains destructive readout after a finite carrier exists. It is not unitary evolution, quantum decoding, or physical deletion. Complete unitary apparatus outputs retain the information omitted from the observer's classical interface.

The completed mathematical release consists of the local channel theorems, the Clifford test, the magic counterexamples, the responding family, the quantitative witness, the control accounting, and the endpoint and recurrence distinctions. No physical field-to-memory receiver, metric response law, gravitational endpoint, or admitted analytic quantum coefficient is established.

Version 2 turns the corrections into tests. A proposal fails if it names a factor ``geometric'' without a dictionary (Theorem), omits any one of the three bridge lines (Theorem), infers leakage from a magic label (Theorem), reads a zero commutator witness as nonresponse (Theorem), reads leakage as a resource measure (Theorem), or takes a heat ledger, a Page history, or a diary restoration from channel data alone (Theorems , ,).

Version 3 reads the proposed cosmological predictions as conditional statements. What survives is a structure of constraints with sharp constants: a probe response is bounded by $4\|O\|\delta_{full}$ (Theorem); it requires a nonscalar compression (Theorem); it can be correlation-only (Theorem); it samples a prescribed profile without backreaction (Theorem); and every step from resource to metric law needs a law that is not supplied here (Theorem).

Declarations

Verification. The source supplies analytical arguments. The finite numerical checks of Appendix are sanity checks of stated formulas, not proofs. No proof-assistant verification, independent technical review, or journal acceptance is asserted.

Authorship and source accessibility. The title page names Ngo Cheung; authorship requires confirmation. The responsible author should confirm archival accessibility of the unpublished comparisons before submission. Missing publication metadata and persistent identifiers are not invented.

Originality and scope. Exact correction, no-hiding, Stinespring dilation, stabilizer structure, and information–disturbance are established theories. The present manuscript does not claim priority for them. Its explicit comparisons do not identify different models, resource conventions, or geometric observables.

appendix

Documentary analytic boundaries

The supplied analytic excerpts concern a different distributional characteristic-composition problem. Their

kernels, carrier, signs, cutoffs, and observable are not identified with *C*, *R*, *B*, *V*, or U_{θ}.

The supplied XIV-H release stops at its H2 transition-amplitude requirement. XIV-I constructs its named internal amplitude module at I2 but does not construct the complete joint characteristic map at I3 [CheungXIVH,CheungXIVI]. XIV-J supplies reconstruction descent and an off-characteristic reference for $K_6 + K_8 + K_9$, not the full six-branch map on tests meeting the characteristic locus [CheungXIVJ].

The necessary algebraic descent condition can be proved independently.

Lemma. Let $Rec: P \to K$ be a surjective linear map. A linear candidate $M: P \to D$ factors uniquely through K if and only if

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{M}.$$

Proof. Necessity follows from factorization. Conversely, define $M(Rec\ p) = M(p)$. Presentations of the same reconstruction differ by a null presentation, so this is well defined and unique.

par *Non-claim.* It does not construct a characteristic product. par

This interface is the one recorded in [CheungXIVJ]. It does not construct a characteristic product. The identity $q\delta^{'}(q) = -\delta(q)$ likewise shows that presentation degree need not descend to an intrinsic symbol. It does not invalidate an already proved internal reconstruction.

The remaining documentary chain is

full joint characteristic map
→ joint section and controlled remainder
→ uniform transverse estimates
→ contact compatibility
→ complete output wavefront theorem
→ admission of the required coefficient.

No arrow is supplied by finite typing, Enter, a resource label, or a residual witness.

Under the supplied contract, the admission-dependent exit would require the unasserted membership

$$Q_H R_1^{(1)} \in \mathcal{E}_H.$$

These symbols identify that separate documentary problem; they are not objects constructed here. XV-B contains abstract lifting and extension interfaces but does not admit the inherited coefficient [CheungXVB]. Thus its admission-dependent exit is not opened by En04-A.

The supplied XVI-N obstruction is conditional on its specified massless instance and route hypotheses [CheungXVIN]. It neither constructs the missing characteristic map nor discharges a universal analytic completion. The corresponding broader obligations are not discharged here.

The analytic continuation is not the parallel relative-classical XIV-A–XIV-B branch. XIII-E's alternative route is not imported. Later continuation or patch labels do not constitute completed maps. These statements report the supplied scope, not universal nonexistence theorems.

In the supplied terminology, O.1 remains NOT_CLOSED. This is a documentary status label, not a new mathematical object closed by the present channel proofs.

OpenAI catalogue: separation, not premise

The supplied companion record attributes the following family descriptions to OpenAI [SuppliedCatalogue]. Their proofs, attribution, and publication status are not independently verified here. No family is a premise or an evaluation of U_{θ}, the four-Majorana control, or a gravitational model.

toprule Family Supplied topic Refused transfer

midrule 227 Sherrington–Kirkpatrick mixing, dynamical transitions, and critical autocorrelations No quantum scrambling, OTOC evaluation, or four-Majorana recovery certificate.

279 Exact quantum factoring over a fixed finite gate set No compilation or complexity theorem for the admitted unitary constants.

274 Moore's parity conjecture for *QAC^0* No limitation on this differently specified typed fragment and no residual-response evaluation.

276 Classical capacity of qubit generalized amplitude-damping channels No complete-complement decoder and no substitute for exact whole-code correction.

265 A two-dimensional area law from a global spectral gap OAI265 Not the leakage history of Theorem ; no geometric dictionary.

272 Entanglement with zero distillable secret key in local dimension ten OAI272 Not the dictionary or resource certificate of Corollary .

280 Strongly rational unitary vertex operator algebras and conformal nets OAI280 Not a certificate; Oai280-A's channels are not *V_1*, *V_2*, *V_3* of Theorem .

bottomrule

In particular, ``Family 227 by OpenAI" is not an estimate used in the exact control proof, and ``Family 279 by OpenAI" does not justify exact synthesis of a controlled continuous-angle rotation. OpenAI catalogue entries remain documentary separations. O.1 stays NOT_CLOSED; no quantum coefficient is admitted; XV-B's admission exit is not opened; and the separate XVI-N obligations are not discharged.

A submission package for a future physical bridge

A positive same-model magic-to-geometric-response claim should archive at least:

• the physical code and complete preparation or purification;

• the field-to-carrier interface and approximation topology;

• the frozen Pauli or other resource convention;

• the complete coherent evolution and access partition;

• the precise magic quantity, free operations, and validated resource certificate;

• the full logical matrix-unit channels, not only populations;

• a residual response witness and its error allowance;

• independently justified geometric observables and matter labels;

• the same-model dynamical or constitutive comparison connecting those observables to geometry;

• a complete quantum decoder or a proved recovery obstruction;

• separate quantum-return and classical-observation programs;

• all syndrome, failure, work, measurement, and discard outputs;

• compatible regulator and implementation allowances;

• a justified common time interval or access-preserving endpoint.

A vacuum magic claim, a non-Clifford gate count, a selected entropy, a classical record, or a finite-program termination theorem does not replace this package.

Version crosswalk, v1 to v2

Parent: En04-A as served by the companion site, sha256 91ebd6ca004f0593 6ecc1c17874007d5 8290c9c3425ec3b4 edd0ec3ed624c2e2; it carries no version marker and is treated as version 1. No v1 label is removed or renamed, and no v1 statement is changed.

toprule v1 item v2 status

midrule Sections 1–9, all labelled results Kept; a non-claim line added after each result.

thm:linearresponse Kept; its non-necessity is now Theorem .

prop:geometrywitness, cor:bridge Kept; classified by Theorems and .

cor:noimplication, prop:independence Kept; quantified by Theorems and .

thm:endpoint, prop:recurrence Kept; separated from Page and record histories by Theorem .

thm:drop, thm:classical Kept; combined in Theorem .

Unlabelled reconstruction-descent lemma (App.) Label lem:descent added; statement unchanged.

v1 Section 12 (ledger), Section 13 (conclusion) Now Sections and ; v2 rows and paragraph added.

New Section : thm:factor, thm:bridge-sharp, lem:cliffdil, thm:magic-fibre, lem:cq, thm:second-order, prop:secondorder, thm:pairs, thm:heat, thm:not-page, thm:drop-exact; Appendices –.

Typesetting textbackslash mathscr now loads mathrsfs; two display blocks ending in textbackslash end{gathered} now close with textbackslash]; the abstract is shortened; the OpenAI table adds Families 265, 272, 280.

bottomrule

Plan verdicts and plan corrections

Verdicts on the eight theorems of the plan: *held* (proved as stated, possibly with scope made explicit), *corrected* (a nontrivial or true form is proved instead), or *rejected*. None is rejected.

toprule Theorem Verdict Proved form

midrule thm:factor corrected (PC-1) Criterion; adjoined factor always blind; on fixed B iff exact recovery from RE ; fails for V_{bad} and prime $\dim B$; single null observable whenever $\dim B > d$.

thm:bridge-sharp corrected (PC-2) Each hypothesis necessary for (C3); (iii) idle for the printed conclusion (C1)+(C2).

thm:magic-fibre corrected (PC-3, PC-4, PC-8) Flag, declared and minimal ($\tau \le 1$) T-count, stabilizer-entropy bins; $d = 2^k$; minimax indicator error $1/2$.

thm:second-order held Explicit K, $m \equiv 0$, $\delta_{full} = 1/2 \sin^2 \lambda$; leading coefficient $1/2$ exact.

thm:pairs corrected (PC-5) Values held; V_2 is the T-dressed copier, not a rotation-family member.

thm:heat held (PC-9 scope) Same carrier, different reservoir records; no heat computed.

thm:not-page held (PC-7 scope) Leakage history does not fix archive, entropy, mouth, or endpoint data.

thm:drop-exact held (PC-6) Cut RK; exact drop; record alone and classical-only error $\ge 1 - 1/d$.

bottomrule

Plan corrections.

• thm:factor: the literal statement is trivial when $\dim G = 1$ or $\mathcal{O}_G$ contains only scalars, and false in the nontrivial full-factor form for V_{bad} and for prime $\dim B$ (Remark).

• thm:bridge-sharp: hypothesis (iii) is not necessary for the printed conclusion of cor:bridge; necessity holds for the magic-accompanied conclusion (C3) (Remark).

• thm:magic-fibre: magic labels need a qubit frame; stated for $d = 2^k$, other d after a declared padding.

• thm:magic-fibre: T-count is a circuit property; it is replaced by the declared circuit count or the minimal count τ. No lower bound on τ above 1 is claimed.

• thm:pairs: in the rotation family, $\delta_{full} = 1/2$ only at the Clifford point $\theta = \pi/2$; V_2 replaced (Remark).

• thm:drop-exact: the plan cites En09-B thm:cDsharp for the sharp return $1 - 1/D$. In the fetched En09-B (v6), $1 - 1/D$ is prop:sharp; thm:cDsharp is the robustness constant $2 - 2/D$. Both are cited as separations only.

• thm:not-page: by prop:recurrence, a finite autonomous Hamiltonian from a vacuum start never has $\delta_{full}(t) \to 0$; the hypothesis concerns a declared family or an access-preserving limit.

• thm:magic-fibre: the error $1/2$ is the minimax value over $[0, 1]$-valued or randomized reports; a deterministic $\{0, 1\}$-valued report has worst-case error 1 on the same fibres.

• thm:heat: ``fixed complete isometry'' is read as a fixed carrier $C \to R \otimes B$. If the ledger registers are adjoined to the residual, the complete isometries differ by a fixed code-independent factor with the same $\mathcal{N}$, δ_{full}, and ε_{rec}^R.

Version crosswalk, v2 to v3

Parent: En04-A-v2, frozen, sha256 6e8ee2fe5fcf3990 579d21ba6dcecdd2 13254b2fbe5b51a6 86080c2d2038434 (MANIFEST ea486e3d4968b6a2 b30eb04cc3dce0c5 9cd305d375b1602c 837c514aac665e17). No v1 or v2 label is removed or renamed, and no v2 statement is changed.

toprule v2 item v3 status

midrule Sections 1–12, all labelled results Kept unchanged.

Ledger, conclusion, status tail One row, one paragraph, and a v3 status line added.

lem:witness Kept; the sharp constant and the normalized form are Theorem .

thm:rotation, lem:cq Kept; reused for Theorems , , .

thm:magic-fibre, lem:cliffdil Kept; extended in both directions by Theorem .

thm:second-order, thm:not-page, thm:drop-exact Kept; read conditionally in Theorems , , .

New Section : def:package, lem:minimax, thm:P14, thm:P2, thm:P12, def:compression, thm:P4, thm:P5, thm:P6, thm:P7, thm:P16, thm:P3, thm:P8, thm:P9, thm:P10, thm:P11, def:layers, prop:layers, def:requirements, thm:chain, conj:18, prop:analogues; Appendices and ; a v3 check table.

Bibliography One entry added: En04-B v2 (unregistered draft), source record; not a premise.

bottomrule

Prediction verdicts, plan corrections, and documentary corrections (v3)

Verdicts: *held* (proved as a conditional theorem on declared packages), *corrected* (a precise true form is proved and the plan's form is recorded as false or loose), *conjecture only* (no proof is offered for the cosmological reading; a finite analogue may be proved). No item is held as a cosmological prediction.

begingroup Verdicts on the plan items of t329u.

toprule Item Verdict Exact statement proved

midrule endfirsthead toprule Item Verdict Exact statement proved

midrule endhead #1 held Leakage without probe response: Prop. (1); equal channel data with different host data: Thm. (c), Thm. .

#2 held $w \neq 0 \Leftrightarrow A_O_G$ nonscalar; $\delta_full \geq |w|/(2s(O_G))$; $\varepsilon_rec\hat{}R > 0$ (Thm.).

#3 held Equal labels with zero, nonzero, and opposite responses; equal response with $M_2 = 0$ vs $log_2\ (4/3)$ (Thm.).

#4 held Constant profile $\Rightarrow O_\{\mathscr{R}\} = \mathscr{R}_0I$, zero witness for every carrier; other probes respond (Thm.).

#5 corrected (PC-11) $w = (\kappa_1 - \kappa_0)sin\hat{}2\ \eta + sin\ 2\eta Re\ c$; the plan's form needs $c = 0$; sampling with $\mathscr{R}\hat{}\{(1)\} - \mathscr{R}\hat{}\{(0)\} \equiv 0$ (Thm.).

#6 held (PC-15) Fixed host datum: δ_full ranges over $[0, a_d]$, minimax error $a_d/2$; fixed declared G: range $[0, 1/2]$, error $1/4$ (Thm.).

#7 held Same leakage history, entropy over $[0, 1]$ bit, minimax $1/2$ bit; host function unconstrained (Thm.).

#8 held (DC-2) Classical floor $1 - 1/d$, attained; distinct from a_d; an exact parameter label does not lower it (Thm.).

#9 held, constants exact (PC-12, DC-3) $F_e = 1/d\hat{}2$, error a_d; success $1/2(1 + d_tr)$; $|\hat{\alpha} - \alpha| \leq |\hat{s} - s(\alpha)|/c$ on a co-Lipschitz interval; witness $c = \sqrt{(2)}$ (Thm.).

#10 corrected (PC-13) $\delta_full = 1/2\sqrt{(1 - |\langle\tau_0|\tau_1\rangle|\hat{}2)}$ ($1/2$ iff orthogonal), $1 - 1/d$ for d orthogonal states; clock = copier complement (Thm.).

#11 held (PC-17) $\delta_full = 1/2sin\hat{}2\ \lambda$, $\varepsilon_rec\hat{}R = 1/2(1 - cos\ \lambda)$, witness $2sin\hat{}2\ \lambda$, first derivative zero (Thm.); Δg, ΔR, $\Delta H \sim \lambda\hat{}2$ not claimed.

#12 held Both marginals $\mathscr{T}_\{I/2\}$, joint nonconstant, $\delta_full = \varepsilon_rec\hat{}R = 1/2$ (Thm.).

#13 held $X_E \otimes X_G$ witness 2, optimal over unit-norm products and equal to $4\|O\|\delta_{full}$; not evidence of a geometric law (Thm.).

#14 corrected (PC-10, DC-1) $|\Delta\langle O\rangle| \le s(O)d_{tr} \le 2s(O)\delta_{full} \le 4\|O\|\delta_{full}$; $\gamma_O \le 2\delta_{full}$; all equalities for the copier with Z_B; plan's $2\|O\|\delta_{full}$ false (Thm.).

#15 held (PC-15) Fixed host: all carriers, $\delta_{full} \in [0, a_d]$; with a supplied dictionary $\mathfrak{D}$, δ_{full} is a function of h (Thm. (b),(d)).

#16 held κ-independent data: minimax $\ge (\sup K - \inf K)/2$, infinite on $(0, \infty)$; injective supplied law identifies κ symbolically (Thm.).

S 17 held as proposition (PC-14) Layers L0–L5 (Def.); each L$k \Rightarrow$L$(k + 1)$ with explicit example; L2⇒L1 (Prop.).

S 18 A–H conjecture only; finite analogues proved Conj. ; analogues A–H in Prop. .

S 19 held as definition and theorem Requirements R1–R10, of which R2, R3, R5, R6 are supplied as declared data (Def.); every arrow fails without, and holds with, its added law (Thm.).

bottomrule endgroup

Plan corrections (v3). 9

• #14: the displayed constant $2\|O_G\|$ is false (copier: $2 > 1$); sharp constants are $2s(O)$ and $4\|O\|$, or $\gamma \le 2\delta_{full}$ with γ normalized by $2\|O\|$ (Remark).

• #5: the witness has the cross term $\sin 2\eta \mathrm{Re}\, c$; the plan's form holds for $c = 0$ (Remark).

• #9: ``$\lesssim$" is replaced by the exact inequality on an interval where s is co-Lipschitz (Remark).

• #10: leakage $1/2$ needs orthogonal qubit clock states (Remark).

• S 17: field response is defined by one-point smeared data; with second moments the example of Prop. (2) would respond. The plan's layers are not independent in reverse: probe response implies residual response.

• #6, #15: the range $[0, a_d]$ needs $\dim R, \dim B \ge d$ (Remark).

• #3: the plan's replacement $\Delta R = \kappa \mathscr{W}[\cdot]$ is not adopted; $\mathscr{W}$ and κ are not supplied, and κ stays a symbol.

• #11 and S 18B: the finite analogue concerns δ_{full}, ε_{rec}^R, and a probe witness, not Δg, ΔR, or ΔH.

Documentary corrections.

• The normalized response γ and $\gamma \le 2\delta_{full}$, the host fibre, and the curvature sampling witness (#4, #5, #6, #14, #15) are En04-B material CheungEn04Bv2, not En04-A v2 theorems; they are proved here locally.

• #8 calls $1 - 1/d$ ``the manuscript's shredding theorem"; in En04-A it is thm:classical and thm:drop-exact (with I-C thm:A CheungIC); the hosted shredder cut belongs to En04-B.

• The echo (#9) and the clock (#10) are not En04-A v2 theorems; they are proved here locally (Theorems ,).

• The plan attributes the sampling witness to ``the manuscript"; En04-A v2 has no profile $\mathscr{R}$ or modes f_a; Definition introduces them.

What stays open. A physical spacetime, field-to-carrier map, dynamical matter–geometry coupling, derived metric response, continuum limit, and distinguishable prediction (R1, R4, R7–R10); a carrier dictionary $\mathfrak{D}$; a sensitivity law fixing κ; a same-model law (L5); every conjecture of Conj. as cosmology. O.1 stays NOT_CLOSED.

Source records consulted

Citations identify source records only; no proof transfer. Each record was fetched read-only from https://enter.grok.me/api/manuscript/<ID> or the catalogue page, on 25 October 2026. The hashes identify the fetched copies.

toprule Record Labels used sha256 of fetched copy

midrule Hawk04-A thm:global, thm:exact, cor:nohiding, thm:four-majorana 68fd298348547eeb dcac8e9e91571fc1 6aa2abecad217090 8a6f03bebf0b3f2d

Hawk04-B thm:drop 9b2456d167c530fd b3b57d706d49ffe3 8538d82128d7e8ed a6400101d13d05f8

Hawk05-B thm:page 0404bd27adc8c80d 1f2e12f860ac60b9 a421349da7826246 f8ca0eb715d3ca4a

I-C (v2) thm:A, thm:B 10f3878a3e890d8d 1b10cbfcfdf681a9 441b642a3baf420a fccbe3be2c464199

I-C-Th thm:access, thm:thermal, thm:reset 3eb1165a52673a06 8a90f0025663edcb 7be3cc4482011439 37832701d4dcf6ef

En07-A prop:no-landauer-from-channel 5b2641e125dec924 3ea5f48fabc48325 2b0614c59682466e caf57fb0d200bde6

Res01-C thm:island, thm:haar, thm:recordgap 5311218b51faedfe 6aea65484ba6b705 7884314bf9fb88da 81bba3f437579916

En09-B (v6) prop:sharp, thm:cDsharp 979843477ff4c3bb a1698a63376c6ee9 ed5c074bb3b03d76 cf46d5a15915c459

Big01-A (v5) thm:fibre 8849b3556e2925be ed6a009218d8d84a 4a2e9abd2c023dd4 64437122de0f8c82

Big04-A-coeff thm:C-adaptive 69d97b6fb3d598e7 31d4a57254c41a4c 189a0e686f2cb00b 62144c2bbaa03e54

Oai280-A prop:goodbad f0aede62e8eb5502 6c6b5b587066ff89 614f29192669e859 eb0bdaf5db0ffc1e

Family 265 catalogue page 265-01 cdfc95803023297f 49a259566e5ae59c db75cb18090193cc 2fafa605869adc15

Family 272 catalogue page 272-01 6ac380c067e17932 cb62b7e7d18db139 af242a6828fbeb69 a6aa32a2bb80ac5c

Family 276 catalogue page 276-01 229548f5c9e1d17a d2e4ce72f9e2c150 1c8fd799b77c8f1e 47c71bef18184490

Family 280 catalogue page 280-01 399604a5114840d0 de5c47cb074ded37 3746391a2321e522 57fc08356e94def1

En04-B not registered (API 404) none

En04-B v2 draft thm:leak-exact (comparison only) d10c2a4c81d90f46 9827d77619a6087b c91bc75a69f5bd08 14aad1f7a14af5a0 (local file)

bottomrule

All listed labels were found in the fetched copies. The I-C labels sit in a mainthm environment. Catalogue hashes are of text extracts of the pages. En04-B is cited only as an unregistered source record; hosting, Green-sector blindness, the rod witness, Hosts A/B, the host-fibre certificate, and the shredder cut are not re-proved here. Version 3 also consulted, read-only, the unregistered En04-B v2 draft supplied in the working conversation, for its normalization of γ and its rod form only; the v3 theorems that overlap it are proved locally.

Finite numerical checks

The checks are finite-dimensional sanity checks of the formulas above: semidefinite programs (solver CLARABEL, tolerance 2×10^{-6}) for half-diamond leakage and optimal recovery, sympy for the symbolic identities, Pauli-spectrum tests for stabilizer states and Clifford unitaries, negative controls, and mutation tests. They are not proofs. Totals: 161 checks, 161 passed, 0 failed (positive: 59, negative-control: 7, mutation: 6, and 89 version-2 checks rerun as regression). The checks file has sha256 6768bfffa1f4645c dfc5fae2df4ceb57 82b42673b57bcbcf eb6bb5a33b5c5a88.

Mutation tests confirm that the following false variants are detected: the constant $2\|O\|$ in Theorem , the witness without the cross term in Theorem , a minimax value $(b - a)/4$, a response that is a function of M_2, the floor $1 - d^{-2}$ for classical records, and leakage $1/2$ for a non-orthogonal clock.

Version 3 checks. toprule Result checks passed

midrule lem:minimax 3 3

prop:analogues 2 2

prop:layers 6 6

thm:P10 7 7

thm:P11 4 4

thm:P12 7 7

thm:P14 13 13

thm:P16 2 2

thm:P3 3 3

thm:P4 2 2

thm:P5 10 10

thm:P6 3 3

thm:P7 1 1

thm:P8 4 4

thm:P9 5 5

bottomrule

Version 2 checks (rerun). toprule Result checks passed

midrule lem:constant(v1) 2 2

prop:independence(v1) 1 1

thm:bridge-sharp 3 3

thm:drop-exact 13 13

thm:factor 17 17

thm:heat 3 3

thm:magic-fibre 23 23

thm:majorana(v1) 4 4

thm:not-page 4 4

thm:pairs 6 6

thm:rotation(v1) 4 4

thm:second-order 9 9

bottomrule

References

E. Knill and R. Laflamme, ``Theory of quantum error-correcting codes," *Physical Review A* **55** (1997), 900–911. doi: 10.1103/PhysRevA.55.900. arXiv:quant-ph/9604034.

S. L. Braunstein and A. K. Pati, ``Quantum information cannot be completely hidden in correlations: Implications for the black-hole information paradox," *Physical Review Letters* **98** (2007), 080502. doi: 10.1103/PhysRevLett.98.080502. arXiv:gr-qc/0603046.

D. Kretschmann, D. Schlingemann, and R. F. Werner, ``The information-disturbance tradeoff and the continuity of Stinespring's representation," *IEEE Transactions on Information Theory* **54** (2008), 1708–1717. doi: 10.1109/TIT.2008.917696. arXiv:quant-ph/0605009.

J. Watrous, *The Theory of Quantum Information*, Cambridge University Press, 2018.

V. Veitch, S. A. H. Mousavian, D. Gottesman, and J. Emerson, ``The resource theory of stabilizer quantum computation," *New Journal of Physics* **16** (2014), 013009. doi: 10.1088/1367-2630/16/1/013009. arXiv:1307.7171.

D. Fattal, T. S. Cubitt, Y. Yamamoto, S. Bravyi, and I. L. Chuang, ``Entanglement in the stabilizer formalism," arXiv:quant-ph/0406168, 2004.

M. Horodecki, P. W. Shor, and M. B. Ruskai, ``Entanglement breaking channels," *Reviews in Mathematical Physics* **15** (2003), 629–641. doi: 10.1142/S0129055X03001709. Preprint title: ``General Entanglement Breaking Channels," arXiv:quant-ph/0302031.

T. Kato, *Perturbation Theory for Linear Operators*, Classics in Mathematics, Springer, 1995. Reprint of the

second edition. doi: 10.1007/978-3-642-66282-9.

C. Cao, ``Non-trivial area operators require non-local magic,'' *Journal of High Energy Physics* 11 (2024), 105. doi: 10.1007/JHEP11(2024)105. arXiv:2306.14996. Cited with its operator-algebra and code hypotheses, not as a generic magic-to-complement-nonconstancy theorem.

C. Cao, G. Cheng, K. Karthikeyan, C. Li, and J. Preskill, ``State-dependent geometries from magic-enriched quantum codes,'' arXiv:2603.13475v1, 2026. Perturbative skewed subsystem codes, proto-area entropy, and tripartite non-local Choi magic. No generic identification with *δ_full* is imported.

M. Moosa, ``Type III von Neumann algebras are magical,'' arXiv:2608.12512v1, 2026. Continuum/thermodynamic-limit comparison only; no finite-carrier residual-response conclusion is imported.

N. Cheung, *Landmark I*, version 2. Supplied unpublished manuscript excerpts, n.d. Source identifiers I.tex and I_v2.tex in the supplied records. In particular, thm:main, ``Characterization.'' Born probabilities and positive-branch destructive residuals within the declared strategy class. https://enter.grok.me/browse/I.

N. Cheung, *En03-A*, version 1. Supplied unpublished manuscript excerpts En03-A.tex, n.d. Minimal linear typed interface and relation to a larger calculus. No field preparation or gate-library universality is imported. https://enter.grok.me/browse/En03-A.

N. Cheung (2026). Hawk04-A. Unitary Information Preservation and Radiation Recovery: A Complete-Complement Formulation of the Black-Hole Information Problem. *Black-Hole Information as an Access Problem: From Complete Unitary Preservation to Conditional Evaporation Certificates*, 1. Source-record locators: thm:global, thm:exact, cor:nohiding, thm:four-majorana. The required channel statements are proved locally. https://enter.grok.me/browse/Hawk04-A.

N. Cheung (2026). Hawk04-B. Enter, Quantum Recovery, and Complete Residuals. *Black-Hole Information as an Access Problem: From Complete Unitary Preservation to Conditional Evaporation Certificates*, 1, 23. https://enter.grok.me/browse/Hawk04-B.

N. Cheung, *Hawk02-E: Coherent Diaries and Complete-Residual Certification in a Regulated Interacting SYK–Scalar Model*, version 1. Supplied unpublished manuscript Hawk02-E-v1.tex, n.d. Regulated architecture, finite-proxy comparisons, source-energy separation, and fixed-proxy recurrence. https://enter.grok.me/browse/Hawk02-E.

N. Cheung, *En02-B*, version 1. Supplied unpublished manuscript excerpts En02-B-v1.tex, n.d. In particular, thm:B1, resource-certified decoder soundness, complete branch maps, and live-name accounting. https://enter.grok.me/browse/En02-B.

N. Cheung, *En02-C*, version 1. Supplied unpublished manuscript excerpts En02-C-v1.tex, n.d. Reference-preserving sequential soundness and the separation of availability, decoder existence, and executable recovery. https://enter.grok.me/browse/En02-C.

N. Cheung, *En02-E*, version 2. Supplied unpublished manuscript excerpts En02-E-v2.tex, n.d. Gaussian packet states and flagged-cutoff comparisons; commuting-state hypotheses are retained. https://enter.grok.me/browse/En02-E.

N. Cheung, *Em03-C*, version 1. Supplied unpublished manuscript excerpts Em03-C-v1.tex, n.d. Joint apparatus interfaces, finite records, coherent comparisons, and physical receiver boundaries. https://enter.grok.me/browse/Em03-C.

N. Cheung, *Landmark XIV-H: Internal kernel relations on the operator route*, version 1. Supplied unpublished manuscript excerpts XIV-H.tex, n.d. Internal maps, regular charts, and the stated H2 stop. Documentary separation only. https://enter.grok.me/browse/XIV-H.

N. Cheung, *Landmark XIV-I*, version 1. Supplied unpublished manuscript excerpts XIV-I.tex, n.d. Constructed I2 internal amplitude module and unresolved I3 joint characteristic map. Documentary separation only. https://enter.grok.me/browse/XIV-I.

N. Cheung, *Landmark XIV-J*, version 1. Supplied unpublished manuscript excerpts XIV-J.tex, n.d. In particular, prop:descent, lem:null, thm:reference, thm:conditionalextension, thm:J2, and cor:noadmission. Supported-sector

references do not admit the full coefficient. https://enter.grok.me/browse/XIV-J.

N. Cheung, *Landmark XV-B*, version 1. Supplied unpublished manuscript excerpts XV-B.tex, n.d. Relative-lifting and extension interfaces; no inherited full characteristic contraction or quantum admission is supplied. https://enter.grok.me/browse/XV-B.

N. Cheung, *Landmark XVI-N*, version r12. Supplied unpublished manuscript excerpts XVI-N.tex, n.d. In particular, thm:massless-obstruction. Conditional specified-instance result, not a universal completion or the missing characteristic-map construction. https://enter.grok.me/browse/XVI-N.

OpenAI, as attributed in the supplied companion record, *Enter Calculus Companion: OpenAI family catalogue excerpts*. Supplied documentary record describing Families 227, 279, 274, and 276; the supplied Family 279 record is labelled rem:oai-279 and dated 7 October 2026. Attribution, proofs, and publication status are not independently verified here. No family is a theorem premise.

N. Cheung, *Hawk05-B: A Finite Page-Curve Derivation under the Enter Discipline*, 15 October 2026. thm:page. https://enter.grok.me/browse/Hawk05-B. Fetched copy sha256 0404bd27adc8c80d 1f2e12f860ac60b9 a421349da7826246 f8ca0eb715d3ca4a. Source record; not a premise.

N. Cheung, *I-C-v2: The Shredder Interface*, October 2026. thm:A, thm:B. https://enter.grok.me/browse/I-C. Fetched copy sha256 10f3878a3e890d8d 1b10cbfcfdf681a9 441b642a3baf420a fccbe3be2c464199. Source record; not a premise.

N. Cheung, *I-C-Th-v1: Three Ledgers for Destructive Readout*, 18 October 2026. thm:access, thm:thermal, thm:reset. https://enter.grok.me/browse/I-C-Th. Fetched copy sha256 3eb1165a52673a06 8a90f0025663edcb 7be3cc4482011439 37832701d4dcf6ef. Source record; not a premise.

N. Cheung, *En07-A: Enter, Reversible Recycling, and Thermodynamic Access*. prop:no-landauer-from-channel. https://enter.grok.me/browse/En07-A. Fetched copy sha256 5b2641e125dec924 3ea5f48fabc48325 2b0614c59682466e caf57fb0d200bde6. Source record; not a premise.

N. Cheung, *Res01-C: Mouth Forms in the Finite Haar Model*, 24 October 2026. thm:island, thm:haar, thm:recordgap. https://enter.grok.me/browse/Res01-C. Fetched copy sha256 5311218b51faedfe 6aea65484ba6b705 7884314bf9fb88da 81bba3f437579916. Source record; not a premise.

N. Cheung, *En09-B-v6: Relational Clock Readout, Sharp Access Limits, and Prescribed Spacetime*, 21 October 2026. prop:sharp, thm:cDsharp. https://enter.grok.me/browse/En09-B. Fetched copy sha256 979843477ff4c3bb a1698a63376c6ee9 ed5c074bb3b03d76 cf46d5a15915c459. Source record; not a premise.

N. Cheung, *Big01-A (version 5): Horizon-Bounded Capacity on a Prescribed de Sitter Host*, 23 October 2026. thm:fibre. https://enter.grok.me/browse/Big01-A. Fetched copy sha256 8849b3556e2925be ed6a009218d8d84a 4a2e9abd2c023dd4 64437122de0f8c82. Source record; not a premise.

N. Cheung, *What the Admitted Data of the Big04-A Host Fix about the Radiation-Label Coefficient* (Big04-A-coeff), 24 October 2026. thm:C-adaptive. https://enter.grok.me/browse/Big04-A-coeff. Fetched copy sha256 69d97b6fb3d598e7 31d4a57254c41a4c 189a0e686f2cb00b 62144c2bbaa03e54. Source record; not a premise.

Oai280-A: Sector-Label Certificates and VOA–Net Separation, version 1, 12 October 2026. prop:goodbad. https://enter.grok.me/browse/Oai280-A. Fetched copy sha256 f0aede62e8eb5502 6c6b5b587066ff89 614f29192669e859 eb0bdaf5db0ffc1e. Source record; not a premise.

N. Cheung, *En04-B*. Not registered on the companion site (/api/manuscript/En04-B returned 404 on 25 October 2026). Cited for thm:blind, the *1 − 1/d^2* quantity, and the hosted shredder cut only to mark them as not used. Source record; not a premise.

OpenAI, as attributed in the companion catalogue, Family 265-01, ``A two-dimensional area law from a global spectral gap," https://enter.grok.me/openai/265-01. Text extract sha256 cdfc95803023297f 49a259566e5ae59c db75cb18090193cc 2fafa605869adc15. Attribution and proofs not verified. Source record; not a premise.

OpenAI, as attributed in the companion catalogue, Family 272-01, ``Entanglement with zero distillable secret key in local dimension ten," https://enter.grok.me/openai/272-01. Text extract sha256 6ac380c067e17932

cb62b7e7d18db139 af242a6828fbeb69 a6aa32a2bb80ac5c. Attribution and proofs not verified. Source record; not a premise.

OpenAI, as attributed in the companion catalogue, Family 276-01, ``Classical capacity and entropy inequalities for generalized amplitude damping," https://enter.grok.me/openai/276-01. Text extract sha256 229548f5c9e1d17a d2e4ce72f9e2c150 1c8fd799b77c8f1e 47c71bef18184490. Attribution and proofs not verified. Source record; not a premise.

OpenAI, as attributed in the companion catalogue, Family 280-01, ``Strongly rational unitary vertex operator algebras and conformal nets," https://enter.grok.me/openai/280-01. Text extract sha256 399604a5114840d0 de5c47cb074ded37 3746391a2321e522 57fc08356e94def1. Attribution and proofs not verified. Source record; not a premise.

R. Landauer, ``Irreversibility and heat generation in the computing process," *IBM Journal of Research and Development* **5** (1961), 183–191. doi: 10.1147/rd.53.0183. Documentary record; source record; not a premise.

N. Cheung, *En04-B v2: Finite Carriers on Prescribed Enter Spacetimes*, draft version 2, 25 October 2026. Not registered on the companion site. thm:leak-exact. Local file sha256 d10c2a4c81d90f46 9827d77619a6087b c91bc75a69f5bd08 14aad1f7a14af5a0. Source record; not a premise.

Status tail

Standing ledger. No metric, field-to-memory receiver, admitted analytic quantum coefficient, or gravitational backreaction theorem is constructed. O.1 stays NOT_CLOSED; no quantum coefficient is admitted; XV-B's admission exit is not opened; and the separate XVI-N obligations are not discharged.

noindentCitations identify source records only; no proof transfer. No quantum coefficient is admitted. Gate 0 and ST4b stay NOT_SUPPLIED; U6-sel is not moved; T3 stays unresolved; no Host C is introduced; no numerical value of κ is given.

Status of version 2: eight plan theorems formed and proved locally (three held, five corrected, none rejected; Appendix). Open: a same-model geometric dictionary; a certified resource in a physical carrier convention; minimal T-count lower bounds above 1; a heat ledger for any physical implementation; a physical receiver, metric response law, Page curve, mouth metric, or gravitational endpoint. This sheet is not registered; authorship requires confirmation.

Status of version 3: sixteen predictions stated as conditional theorems (thirteen held, three corrected; plan corrections PC-10 to PC-17), the layer proposition and the constraint-chain theorem proved, eight candidate phenomena kept as conjectures with finite analogues (Appendix). No cosmological prediction, Einstein equation, numerical κ, or observational claim is made; open items are listed in Appendix .